DOCUMENT RESUME

ED 159 480 CE 018 124

AUTHOR	Carney, Richard
TITLE	Your Electronic Hand Calculator. How to Get the Most Out of It.
INSTITUTION	Rutgers, The State Univ., New Brunswick, N.J. Curriculum Lab.
SPONS AGENCY	New Jersey State Dept. of Education, Trenton. Div. of Vocational Education.
PUB DATE	Sep 78
NOTE	232p.
AVAILABLE FROM	New Jersey Vocational-Technical Curriculum Laboratory, Building 4103, Kilmer Campus, Rutgers University, New Brunswick, New Jersey 08903 ($5.50 plus postage)
EDRS PRICE	MF-$0.83 HC-$12.71 Plus Postage.
DESCRIPTORS	Calculation; Electronic Equipment; *Equipment Utilization; Manuals; *Mathematics Instruction; Postsecondary Education; Secondary Education
IDENTIFIERS	*Calculators

ABSTRACT

Designed for use by students at beginning high school through adult levels, this manual contains seventeen lessons on uses of the electronic hand calculator. Within each of the eight sections lessons presented include objectives, procedural information, examples, and problems to be solved. Following an introductory lesson to the calculator, section 2 provides lessons on three fundamental operations: addition and subtraction, multiplication and division, and powers and roots. Section 3 lessons are on decimals and percents: rounding off numbers, fractions to decimals, and percentage. Two lessons on special features are given in section 4: multiplication and division by a constant, and memory. Section 5 covers order of operations: basic operations and parentheses. In section 6, special keys are presented: key, negative numbers and sign-change key, reciprocal key, and exchange keys. Section 7 has two lessons on the presentation of numbers: significant figures and scientific notation. Review problems are provided in section 8. Selected answers to problems presented in the various sections are appended, and an index is provided. (JH)

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ED159480

State of New Jersey
Department of Education
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YOUR ELECTRONIC HAND CALCULATOR
-- How To Get the Most Out of It

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Curriculum Laboratory
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September 1978

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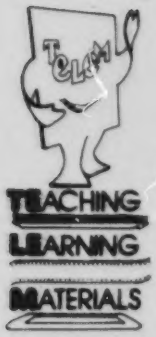

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PREFACE

My first contact with the electronic hand calculator happened during the fall of 1973. One of my colleagues had purchased one that he was using for the calculation of students' grades. I borrowed it a few times and experimented with it. I can remember thinking that it was an interesting device. Strange that this everyday object that nearly everyone owns today was so new just a few short years ago!

The following year, 1974, as school started, the first topic in our Technical Mathematics class was, as usual, the use of the slide rule. From the beginning I noticed undercurrents of displeasure among the students. It didn't take long before the questions started coming. "Do we have to use a slide rule? May we use our calculators instead?" My first concern was to find out how many students had one. To my surprise I found that only four persons in the room did not have access to a calculator. As one of the four persons, I decided to buy one that weekend. That same weekend I put my slide rule in my desk drawer. I suppose some day my son will find it and say something like, "Dad, what is this funny looking stick with the little numbers on it?"

Today the electronic calculator is accessible to practically everyone. Even highly complex scientific calculators can be bought for around $50 today, and there are many models priced as low as $7.50. These models can do just about any calculation that the average person needs to do in everyday living and on the job.

The purpose of this text will be to present some of the basic possibilities of the calculator. Hopefully the reader will acquire a solid foundation and will be comfortable in the use of his/her instrument. This book does not attempt to teach mathematics, but to simply place a person in control of his/her calculator. Once in control, the user will find many new ways to use it – on the job, at a desk working out personal finances, even in the kitchen.

I wish to thank Mr. Benjamin Shapiro, former director of the Vo-Tech Curriculum Laboratory at Rutgers, Dr. Philip Edgecomb, present director, Mrs. Marion Shapiro, editor, and my family Gayle, Lauren, and Glenn for their help and support in this project.

Good Luck, and by all means, keep your batteries charged!

The Author

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1 – INTRODUCTION

Lesson 1 — Getting Acquainted

Objectives:

1. You should be able to identify the two different types of calculator logic.

2. You should be able to define the words *user, keyboard, data keys, function keys, registers, display register, storage register, memory register, floating decimal point, battery-saver or fade circuit, overflow or error indication, "click-stop" keys,* and *readability of the display.*

3. You should be able to name the three types of batteries used in hand calculators.

Information:

The electronic hand calculator is one of the most exciting tools in the history of mathematical computation. It is far superior to any other hand-calculating device that has ever been invented. It is remarkable for its small size, speed, accuracy, and low price.

Electronic hand calculators use two different types of logic. The type generally used by the inexpensive calculators is **algebraic logic.** One of its major advantages is the ease with which a person can learn to use this type of calculator. In algebraic logic the entries are made in the same order that is used in arithmetic and algebra. This is the order in which we normally express a problem.

Example:

Problem: 2 + 3 = ?

Keys to press on your calculator

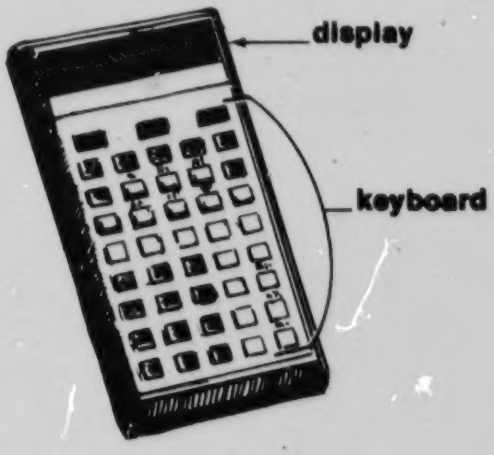
[2] [+] [3] [=]

(The answer, 5, appears in the display.)

The other type of logic is **Lukasciewicz** (sometimes referred to as reverse-Polish). It is used mainly in some scientific calculators. The major disadvantage of this logic is that the entries are not made in exactly the order we would expect from our knowledge of arithmetic and algebra. Therefore, our initial use of the calculator requires a little more effort.

The major advantage of this type of logic is that, once it is mastered, the number of key strokes is often less when solving long, difficult problems.

In this text we will only deal with calculators that use algebraic logic.



At this point we shall start learning the language of the calculator.

The **user** is the person who is operating the calculator.

The **keyboard** is the set of keys or buttons which the user pushes in order to operate the calculator. Each of these keys can be classified as either a **data key** or a **function key**.

"Data" includes all the numbers we are dealing with in a problem. The **data keys** are those which are used to enter, change, or erase the numbers we are dealing with.

Examples:

- [0] through [9] – Digit keys
- [.] – Decimal point
- [π] – pi (the number 3.14159...)
- [EE] – Enter exponent. (In the number 3.5^4 the 4 is the exponent.)

[EE↑] – Move the decimal point and change the exponent
[EE↓] accordingly.

[+/-] – Change-sign key (For example, will change 2.05 to -2.05 and vice versa.)

[C] – Clear or erase numbers in the display register and the storage register.

[CE] – Clear or erase the last entry in the display register.

[CM] – Clear memory.

The function keys are those which actually work the problems. Some function keys move the data from one register to another.

A calculator may have a few function keys or many. There is some variation also in the way these keys are printed, and sometimes in the way they are used. For example, there are different types of "memory" keys and "constant" keys.

Examples:

When we press this key:	*We are telling the calculator – –*
[+]	– Add the next number.
[-]	– Subtract the next number.
[X]	– Multiply the next number.
[÷]	– Divide by the next number.
[%]	– Change number internally from a percent to a decimal (e.g., 6% to .06).
[=]	– Equals, i.e., gives the result of your calculations.
[X^2]	– Square the number in the display.
[$\sqrt{x}$]	– Give the square root of the number in the display.
[X↔Y]	– Switch data in the display register (X) and the storage register (Y).
[X↔M]	– Switch data in the display register (X) and the memory (M).

K	– Keep adding, subtracting, multiplying by, or dividing by the same number. (The constant key.)
M+	– Add the number in the display to the number in the memory.
M-	– Subtract the number in the display from the number in the memory
RCL or RM	– Recall memory. That is, place a copy of the number stored in memory into the display, to become the next piece of data in the problem.

Scientific calculators and special-use calculators have many additional function keys. You may not know what they mean. Your teacher will explain them.

$\bar{x}\,\sigma$	– Mean – standard deviation
$1/x$	– Reciprocal
X!	– Factorial
$\sqrt[x]{y}$	– The x root of y
y^x	– The base y to the x power
Σ	– Add the number in the display to the number in memory.
D/R	– Convert from degrees to radians and vice versa.
sin, cos, tan	– Trigonometric functions
arc, INV, $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$	– Inverse trigonometric functions
log	– common logarithm
lnX	– natural logarithm

[e^x] – natural antilogarithm

[hyp] – hyperbolic functions

[(] [)] – parenthesis – used for grouping purposes to insure proper order of operations.

[STO] – copy the data in the display into the memory location. Any number that previously occupied the memory location is destroyed.

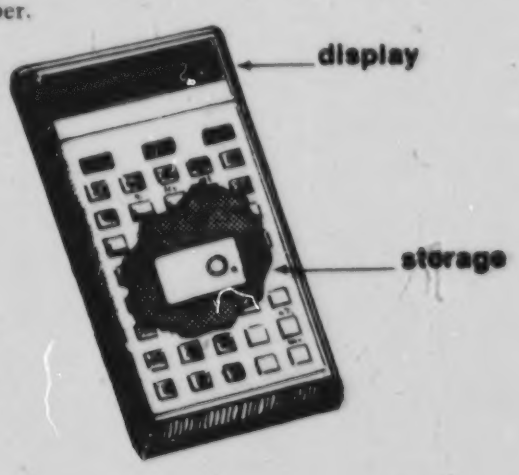
[F] – Used on calculators which double-up on certain keys.

Most of the digit keys above have two meanings. They are data keys if pressed directly, and they are function keys if preceded by the [F] key.

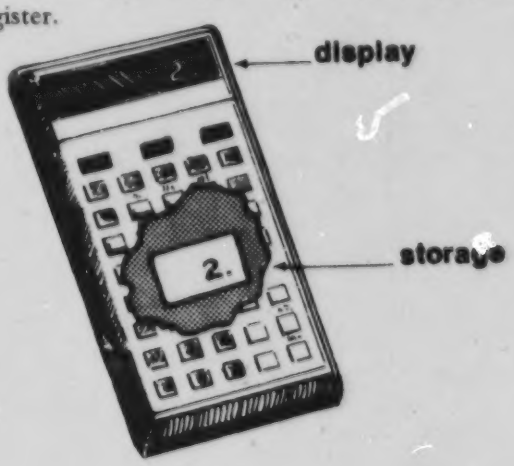
On some calculators you must press several keys (2 or 3 in order, forming a special code) in order to make a certain function take place.

The locations in the calculator where numbers are placed for processing are called **registers**. The three types of registers we will be discussing in this book are the **display register**, the **storage register**, and the **memory register**. The display register is the easiest to keep track of, because the number in this register can be seen directly in the display of the calculator. It is often referred to as the X register.

The number in the storage register (Y register) cannot be seen. It is within the calculator. In the problem 2 + 3 = ?, we first enter the 2, which shows up immediately in the display register with a decimal point after the number.

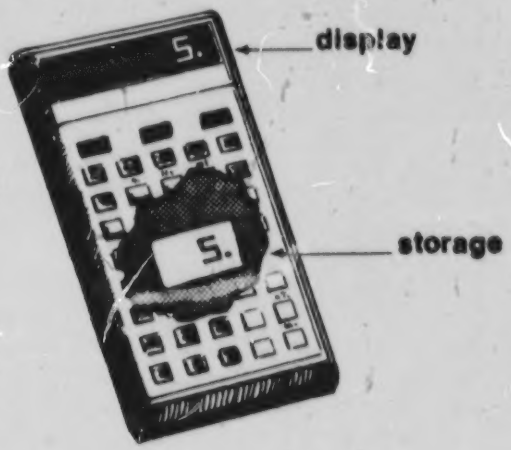


Next we press the [+] key. A copy of the display register is made in the storage register.



We now enter the 3, which has the effect of erasing the 2 in the display.

Pressing the equal sign causes the addition operation to take place between the numbers in the display and storage registers. The answer 5 is now placed in both of these locations.*



Therefore, the storage register is a location that is used to store a number that is about to be used for the next operation (+, −, X, ÷). It may also be used for the storage of the answer to that operation (check footnote). All calculators *must* have display and storage registers.

* In some calculators, the answer (5) is placed only in the display, while the previous entry (3) is placed in the storage register. This condition can be checked by pressing the exchange key [x/y] or [X↔Y] if your calculator has one. Users of this type of calculator will find some variation in the storage register from the figures given in this book.

The memory register (**M**) is present in most but not all calculators. This register is used to store any number that the user may wish, once it is on the display register. Many calculators also allow us to add to or subtract from the memory register by using the [M+] or [M−] or [Σ] keys. When the proper key is pressed, the number is copied from the display register into the memory register without affecting the display and storage registers.

Other features

Most calculators use a **floating decimal point**. This means that the user can enter the decimal point wherever it appears in a number. Also the calculator will automatically place the decimal point in its proper place in the answer to any problem.

On some machines it is also possible to pre-set the decimal point so that all results are rounded off to a certain number of decimal places, generally two or four. Machines designed specifically for dealing with money can be set so that they automatically read in dollars and cents; you don't have to enter the decimal point unless you choose to do so.

In this book we will deal only with floating decimal points.

Calculators have ways of telling you when they cannot do a problem. There are **error indications** and **overflow indications**. You will see these if--

(a.) An entry or calculation result is too big or too small (negative) to fit on the display panel. (Try multiplying 59,000 by 68,000.)

(b.) The user tries to take the square root of a negative number. (Try it.)

(c.) The user tries to get an inverse trig function for an impossible value.

(d.) The user tries to divide by zero.

For example, on the Texas Instruments SR-50, the entire display flashes. On the Texas Instrument TI-1200, the number 0 flashes in the units column. On the Texas Instrument SR-11 a symbol that resembles a capital "C" is displayed. On the Unisonic 1548, the entire display is filled with

zeros followed by decimal points. On the Kings Point SC-20, a symbol Γ appears. On Sharp calculators, a symbol Ε appears. And so on. On each of these calculators an error indication is canceled by pushing the clear key. Different calculators have different ways of handling the overflow condition.

The **battery-saver or fade circuit** is a feature designed to save battery power. If the user leaves a number in the display for more than a certain number of seconds (depending on the model), then the calculator automatically turns off the display except for the last digit, thus saving battery power. For example, the Texas Instruments SR-11 has this feature. To recall the entire number to the display, the [+/-] key is pressed twice. On the Kings Point SC-20, the [=] key is pressed.

The electronic hand calculators can use several different types of batteries. Some of the most frequently used batteries are the carbon-zinc battery, the alkaline battery, the nickel-cadmium battery, and the silver oxide battery. The carbon-zinc, alkaline, and silver oxide batteries are not made for recharging.

The best batteries to use are the nickel-cadmium (NICAD). These batteries are definitely rechargeable. They can be recharged over and over again for quite a long period of time.

The author has compiled the following list of questions to help guide the user in learning about the batteries in his/her calculator. The answers to these questions are usually found in the owner's manual.

Power-Source Questions

1. What type of battery is used?
2. How long will the batteries last?
3. Where can you obtain replacements?
4. How much do replacements cost?
5. Are the batteries rechargeable? Alkaline or ordinary carbon-zinc batteries cannot be recharged.
6. Does the calculator have a provision for recharging the batteries? If so, how is it done? How long does it take?
7. Can the calculator be used while the batteries are being charged?

8. Are there any other precautions to take while charging the batteries?

9. If the calculator is operated with batteries with a low charge, will this cause incorrect entries or calculations?

10. Does the calculator have provision for use of an AC adapter?

11. What is the voltage of the AC adapter? (110V, 220V, 240V)

12. Is the AC adapter also the battery charger?

13. What is the procedure for using the AC adapter?

14. If the calculator is using nonrechargeable batteries, must they be removed during AC operation?

15. If the calculator is using rechargeable batteries, do the batteries *have to be in position* when using the AC Adapter/charger?

Buying a calculator

The decision concerning which calculator to buy should be based upon a person's needs. An engineer or technical student will certainly use different keys than someone who simply balances a checkbook or calculates the amount of discount at a department store sale. Reviewing the list of data keys and function keys presented in this lesson might save a person from having to buy a second calculator within a short period of time. Consulting with a teacher or someone else familiar with mathematics and calculators is usually an excellent idea.

Two practical points about calculator design which are very important to look for are the "click-stop" keys and the readability of the display. The **click-stop** key acts exactly as its name implies. When you press the key it clicks and then stops. With this type of key a person can feel and hear when the key has been depressed properly. There is little chance of error because of an improperly pressed key.

The **readability of the display** is important for the user's convenience. It affects the ease with which the user can read the numbers. Be sure that the display is large and bright enough and is so placed that it can be comfortably read while sitting at a desk.

Questions:

1. Name the two different types of logic that are used in electronic hand calculators.

2. The presentations in this book will deal with the _____ logic only. State the major advantage of this type of logic.

3. Explain what is meant by each of the following:

   a. user
   b. keyboard
   c. data keys
   d. function keys
   e. registers
   f. display register
   g. storage register
   h. memory register
   i. floating decimal point
   j. battery saver or fade circuit
   k. overflow or error indication
   l. click-stop keys
   m. readability of the display

4. If you already have a calculator, then how does your instrument display an overflow? an error indication? How can you continue if you have an overflow?

5. Does your calculator have a battery saver or fade circuit? How can you restore the display?

6. What are four types of batteries used in the electronic hand calculators? Which are made for recharging and which are not?

7. If you have a calculator, then answer all the questions presented in this lesson under the heading "Power Source – Questions," page 9 . The answers to these questions are usually found in your owner's manual.

8. Considering your school courses and plans for your future trade or career, prepare a list of your probable needs for a calculator. Consult with your instructors if possible.

9. Compare this list with the data keys and function keys presented in this lesson. Which keys would it be good to have on your calculator?

10. See if you can add to the list of data keys and function keys presented in this lesson by checking in a store where calculators are sold.

11. For floating-decimal point calculations, enter the following numbers on your calculator. Press the clear key after each entry. Mark any you are doubtful about and check them in the Answer section later (page 211).

a. 25	h. 0.13	n. 5905	u. 6750
b. 25.00	i. 0.013	o. 59.05	v. 675.0
c. 2.05	j. 0.0130	p. $19.36	w. 0.66
d. 0.25	k. 130	q. $19.30	x. 38,000
e. 250	l. 13,000	r. $1900	y. $41.74
f. 2500	m. 13.9	s. 0.0025	z. $41.70
g. 13.10		t. 0.0020	

II – FUNDAMENTAL OPERATIONS

Lesson 1 — **Addition and Subtraction**

Objectives:

1. You should be able to use your calculator to perform addition and subtraction problems.

2. You should be able to clear:

   a. the storage register and the display register
   b. the display register only

3. You should be able to correct an incorrectly pressed function key (addition or subtraction).

4. You should know whether or not it is necessary to clear the registers before starting a new problem.

5. You should be able to define the terms *binary operation, chain addition and subtraction, destructive read-in, commutative property of addition,* and *simulation table.*

6. You should be able to complete simulation tables of addition and subtraction problems.

Information:

A binary operation is an operation that takes place between two elements (parts). Addition, subtraction, multiplication, and division are examples of binary operations. In performing a binary operation with a calculator, one of the elements will be in the display register and the other element will be in either the storage register or the memory register.

Chain addition and subtraction problems are those where the operation of addition and/or subtraction appears more than once.

Example:

$$2 + 3 - 7 + 6 - 5 = ?$$

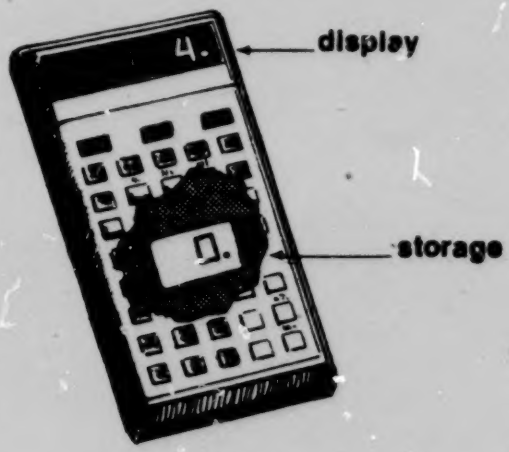
We cannot have two numbers sharing the same register. Therefore, if a number is placed in a register, the number formerly occupying that

position can be thought of as either having been destroyed or removed from that register. Computer people call this condition **destructive read-in.**

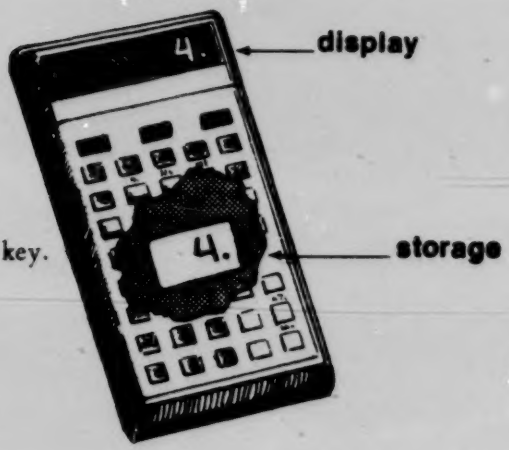
Example:

4 + 6 = ?

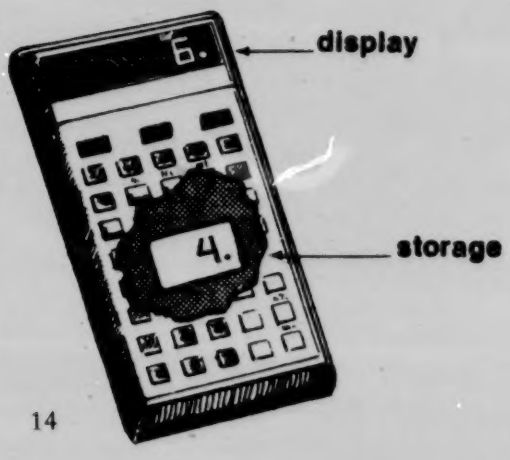
Step 1 – Enter 4



Step 2 – Press [+] key.

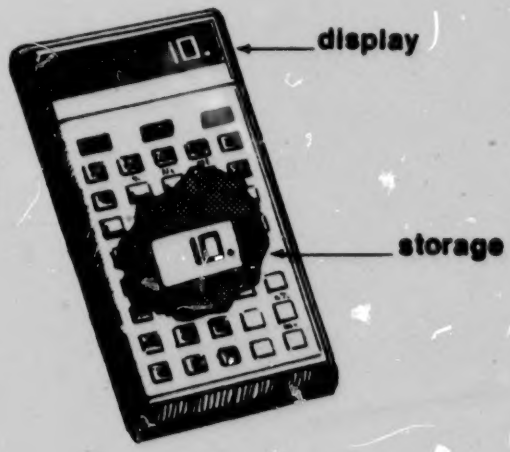


Step 3 – Enter 6



Notice that when the 4 was placed in the storage register, it removed all trace of the 0 which originally occupied that position. Also the 6 replaced the 4 in the display register.

Step 4 – Press [=]



The answer 10 has been placed in both the display register and the storage register, completely eliminating any trace of the 6 and the 4. (But see footnote on page 7).

The word "commutative" means having parts that can be combined in any order and still give the same result. (Think of the word "commute" – to travel back and forth.) The **commutative property** of addition states that $a + b = b + a$. This means that the order in which we add the terms will not change the answer ($a + b = b + a$).

Examples: $2 + 3 = 3 + 2 = 5$

$7 + 3 + 6 = 3 + 7 + 6 = 6 + 7 + 3 = 6 + 3 + 7 = 16$

Subtraction is *not* commutative. If we interchange the two numbers in a subtraction problem, we will not get the same answer. $a - b \neq b - a$

Example: $7 - 5 = 2$

$5 - 7 = -2$ (a different number)*

Therefore $7 - 5 \neq 5 - 7$

*The concept of negative numbers will be discussed in Chapter VI, Lesson 2. For now we can just point out, that, *if we keep our signs straight*, we can do subtraction problems correctly even in reverse order.

Example: $7 - 5 = 2$

$-5 + 7 = 2$

If we have a chain of numbers involving addition and subtraction, however, we can change the order in which we perform the operations and still get the correct answer.

Try these different ways of combining the same numbers:

3 + 4 - 6 + 9 =
4 + 3 - 6 + 9 =
4 - 6 + 9 + 3 =
9 + 3 + 4 - 6 =

You should have got the answer 10 each time.

An extremely valuable exercise in gaining control of the calculator is completing simulation tables. This is the way computer programers become familiar with their computers, and it will help you to become familiar with your calculator.

The word "simulation" in this sense means showing a process that you cannot see directly. The simulation table will allow us to examine on paper all of the steps that are used in solving a problem with our calculator. The first column will be called the Entry column. In this column we will record the data and function keys as they are pressed. The second and third columns will show us the contents of the display register and the storage register. Later in the book we will add another column for the memory register.

ENTRY	DISPLAY	STORAGE	MEMORY

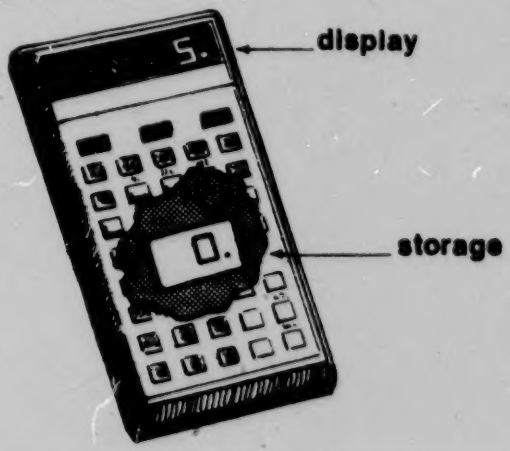
Always use your calculator to check the simulation tables presented as examples in this book.

As mentioned in the first chapter, the calculators we will be discussing all use algebraic logic. We will enter the numbers and functions in the same order that we say them.

Example: Addition

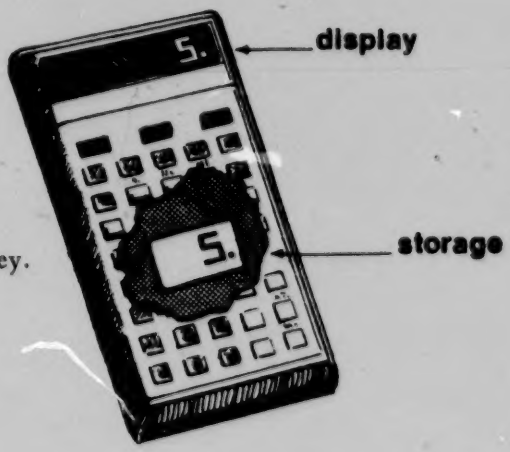
$5 + 7 = ?$

Step 1 – Enter 5



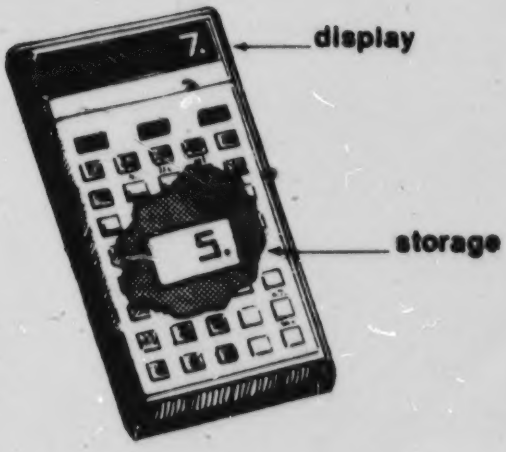
The 5 now occupies the display register.

Step 2 – Press [+] key.



The number in the display is duplicated in storage.

Step 3 – Enter 7



The 7 replaces the 5 in the display register.

Step 4 – Press [=]

(or 5. on some machines)

The answer 12 is placed in both the display and the storage registers.

If this same problem were to be illustrated in a simulation table it would look like this:

5 + 7 = ?

5 + 7 = ?

ENTRY	DISPLAY	STORAGE
5	5.	0.
[+]	5.	5.
7	7.	5.
[=]	12.	12. (7)*(0)*

Example: Chain addition

5 + 12 + 18 + 9 + 31 = ?

ENTRY	DISPLAY	STORAGE
5	5.	0.
[+]	5.	5.
12	12.	5.
[+]	17.	17. (12.)**
18	18.	17.
[+]	35.	35. (18.)**
9	9.	35.
[+]	44.	44. (9.)**
31	31.	44.
[=]	75.	75. (31.)**(0.)**

Notice that each time the addition key [+] was pressed, the total *up to that point* was shown in the display.

*Many calculators act differently in the storage area when the function keys are pressed. The author has found the "0" and "7" to be popular alternatives to the "12" in this problem. The student should check his/her own calculator by using the [X↔Y] or [X/Y] key if the calculator has one. This key interchanges the numbers in the display and storage register and enables the student to see what is in the storage of his/her own calculator.

**Alternative readings in some calculators, as detailed in the previous footnote.

Notice also that, each time a function key ([+] or [=])was pressed, the figure displayed was equal to the sum of the previous figure in the display and the previous figure in the storage register.

Example: Subtraction

13,925 - 6357 = ?

ENTRY	DISPLAY	STORAGE		
13925	13925.	0.		
[-]	13925.	13925.		
6357	6357.	13925.		
[=]	7568.	7568.	(6357.)	(0.)

Example: Chain subtraction

12,573 - 4978 - 256 = ?

ENTRY	DISPLAY	STORAGE		
12573	12573.	0.		
[-]	12573.	12573.		
4978	4978.	12573.		
[-]	7595.	7595.	(4978.)	
256	256.	7595.		
[=]	7339.	7339.	(256.)	(0.)

Again, notice that each time the subtraction key [-] was pressed, the result *up to that point* was shown in the display. Also, pressing a function key ([-] or [=]) operated on the previously displayed number and the number in the storage register.

Example: Addition and subtraction

127 - 49 + 5732 - 2570 = ?

ENTRY	DISPLAY	STORAGE
127	127.	0.
[-]	127.	127.
49	49.	127.
[+]	78.	78. (49.)
5732	5732.	78.
[-]	5810.	5810. (5732.)
2570	2570.	5810.
[=]	3240.	3240 (2570.) (0.)

Notice how pressing the function keys produced *the total to that point* in the display. In each case the previously displayed number was combined with (added to or subtracted from) the number in the storage register.

Example: Changing the order of the entries in the previous problem.

-2570 + 5732 - 49 + 127 = ?

ENTRY	DISPLAY	STORAGE
[-]	(-)0.	0.
2570	(-)2570.	2570. (0.)
[+]	-2570.	-2570.
5732	5732.	-2570.
[-]	3162.	3162. (5732.)
49	49.	3162.
[+]	3113.	3113. (49.)
127	127.	3113.
[=]	3240.	3240. (127.) (0.)*

*From now on the alternative storage readings will not be shown. Most readings shown will be those used in the Texas Instruments SR-50 and similar models.

Notice that, when we changed the order of the terms, we kept the sign that was in front of each one. That was why we got the same answer as we got before.

Many models have a [C] key, meaning **Clear** and a [CE] key meaning **Clear Entry**. Clearing means erasing the number that is in a register and setting that register equal to zero. If we wish to clear the display register and the storage register at the same time, then we press the clearing key [C]. On some models there is a single key [C/CE]. This must be pressed *twice* to get both display and storage registers back to zero.

Example:

ENTRY	DISPLAY	STORAGE
	9.	9.
[C]	0.	0.

If we wish to clear the display register only, press the [CE] Clear Entry key (or the [C/CE] key *once*). This key is extremely valuable. It allows us to correct any mistake we have made in the last number we have entered.

	ENTRY	DISPLAY	STORAGE
Mistake! We wanted 565	562	562.	29.
	[CE]	0.	29.
	565	565.	29.

Notice that the [CE] does not disturb the number in storage.

Occasionally we may press an incorrect [+] or [-] key, and need to correct it. You know that adding zero to a number or subtracting zero from a number will give you the number itself. (a + o = a and a - o = a). We can use this fact to correct an incorrectly pressed [+] or [-] key.

Example: $5 + 2 = ?$

	ENTRY	DISPLAY	STORAGE
	5	5.	0.
Mistake	[-]	5.	5.
	0	0.	5.
Correction	[+]	5.	5.
	2	2.	5.
	[=]	7.	7.

or $5 - 0 + 2 = 7$

Notice that by subtracting zero after the incorrect key [-] was pressed, we did not lose the value in the storage register. In other words, we did not have to start the problem all over again from the beginning. Here is another example:

$3 + 7 - 2 + 4 = ?$

	ENTRY	DISPLAY	STORAGE
	3	3.	0.
	[+]	3.	3.
	7.	7.	3.
	[-]	10.	10.
	2	2.	10.
Mistake →	[-]	8	8.
	0	0.	8.
Correction	[+]	8	8.
	4	4.	8.
	[=]	12.	12.

or $3 + 7 - 2 - 0 + 4 = 12$

23

Most calculators will automatically insert the zero entry for us if we press the [-] key immediately after the [+] key or vice versa. Therefore, it is not necessary for the user to make the zero entry (as was done in the two preceding examples).

Example: 3 + 7 = ?

	ENTRY	DISPLAY	STORAGE
	3	3.	0.
Mistake →	[-]	3.	3.
Entry of zero not necessary →	[+]	3.	3.
	7	7.	3.
	[=]	10.	10.

or 3 - 0 + 7 = 10

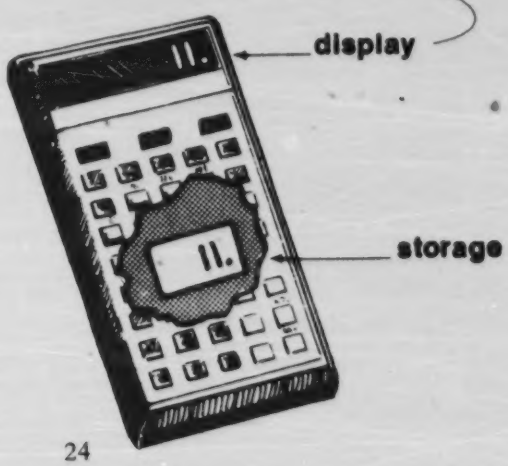
Therefore, we can simply state that the second key ([+] or [-]), will cancel the first.

Examples: [5] [+] [-] [2] [=] 3

[7] [-] [+] [4] [=] 11

Suppose we have just worked out a problem and the answer to that problem is in both the display and storage registers.

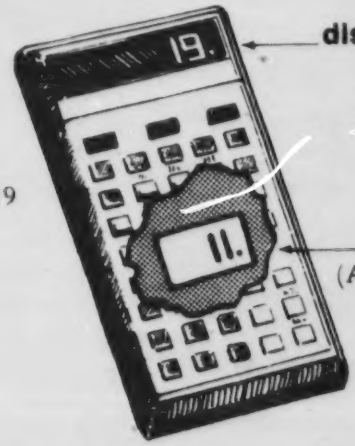
Example: 5 + 6 = 11 Answer



If we decide to start another problem at this point, can we proceed without clearing the registers?

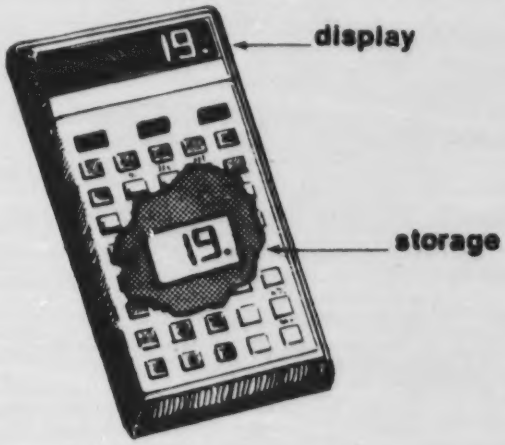
Example: 19 + 24 = ?

Step 1 – Enter 19



Because of the principle of destructive read-in, the 19 completely replaced the 11 in the display.

Step 2 – Press [+]



The 19 is duplicated in the storage register. The 11 in the storage register is destroyed.

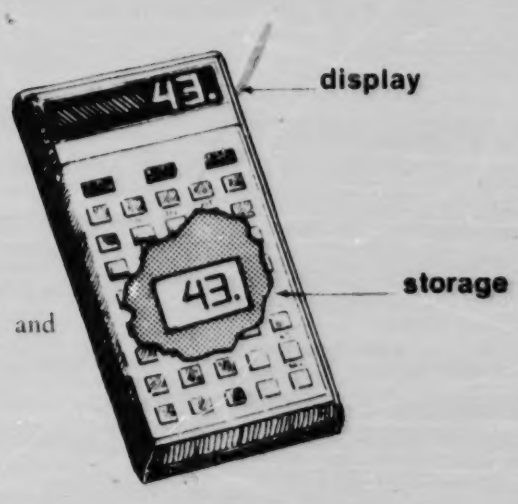
Step 3 – Enter 24

Step 4 – Press [=]

Therefore, we can conclude that it is not necessary to clear the registers before starting another problem *if our first entry is a positive number.**

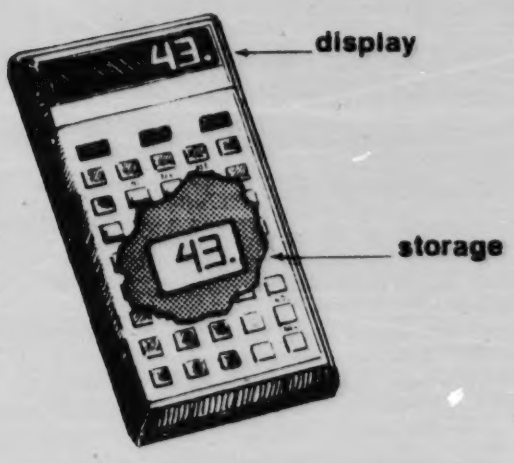
Let's examine what would happen if we were to start a problem where the first entry is preceded by a minus sign.

* This applies only if the previous answer was obtained by pressing the [=] key. If for any reason the previous problem was not ended with the [=] key, then it is necessary to clear the registers before starting a new problem.



The 43 is still in the display and storage registers.

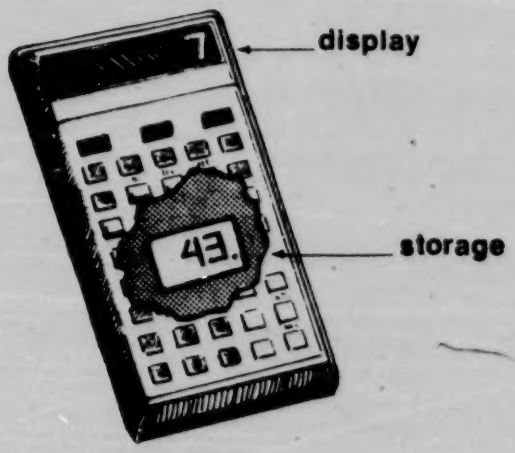
Calculate: - 7 + 3 = ?



Step 1 – Press [-]

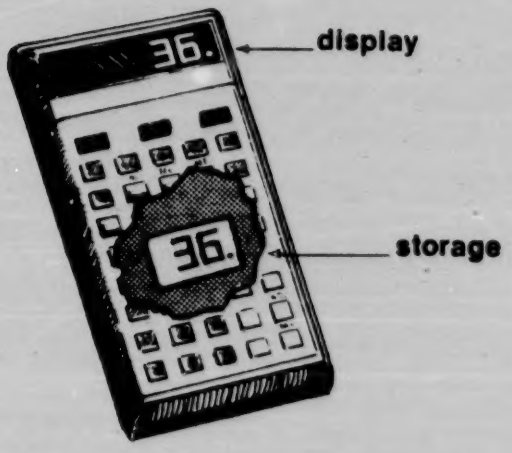
The calculator acts as if this is going to be a continuation of the previous problem.

Step 2 – Enter 7



Step 3 – Press [+]

The calculator displays the answer to 43 - 7 = 36.



The conclusion is that we cannot start a problem where the first entry is the [-] key *unless we clear both registers.*

Entering figures

Because the "floating" decimal point is automatically placed correctly in the answer, you do not have to be concerned about entering final zeros *after* a decimal point. For example, if you were adding this column of figures – –

$129.18
325.70 ← This figure could be entered as 325.7
19.09
56.00 ← This figure could be entered as 56

Try adding the figures both ways. What did you get?

Of course you must be careful when the zeros come at the beginning of a number. They must be entered.

.0079 0.0079	Both these numbers are entered the same way: Point, 0, 0, 7, 9.

Problems:

1. Explain each of the following:

   a. binary operation
   b. chain addition and subtraction
   c. destructive read-in
   d. commutative property of addition
   e. simulation table

2. How do you clear the storage register and the display register in your calculator?

3. How do you clear the display register only?

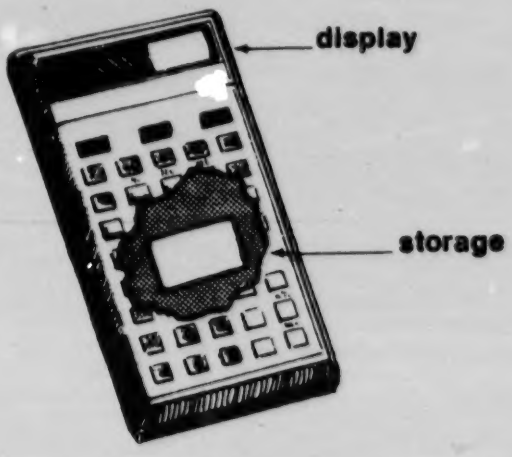
4. Press the following sequence of keys and see if you get the indicated answer. If you do not, then check your owner's manual to find out how to correct an incorrectly pressed [+] or [-] key. If no direct way is indicated, enter a zero and continue with the correct function, as explained above.

   a. [3] [+] [-] [2] [=] 1
   b. [5] [-] [+] [2] [=] 7

   answers

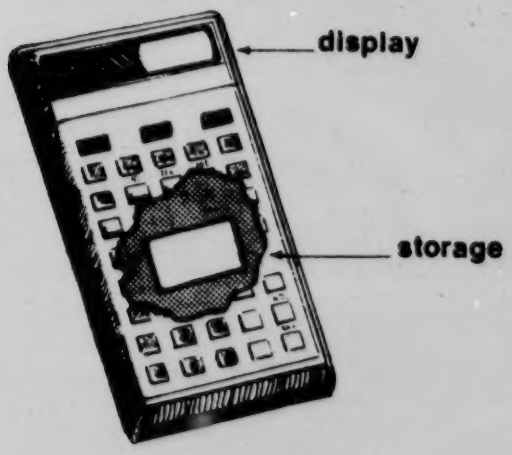
5. Fill in the display register and the storage register for each of the following.

   a. 5 + 6 = ?

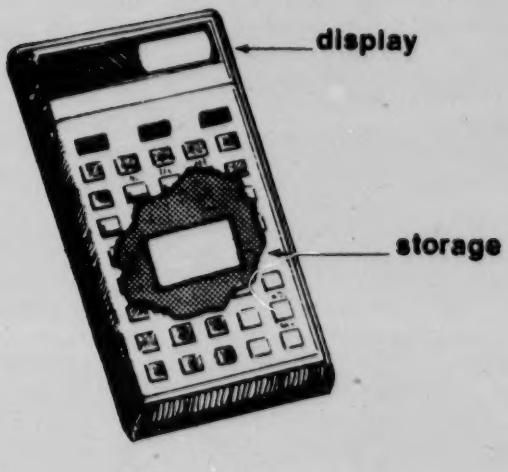


Step 1 – Enter 5

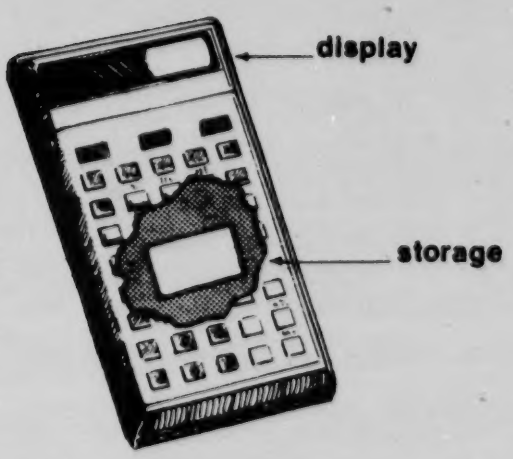
Step 2 – Press [+]



Step 3 – Enter 6

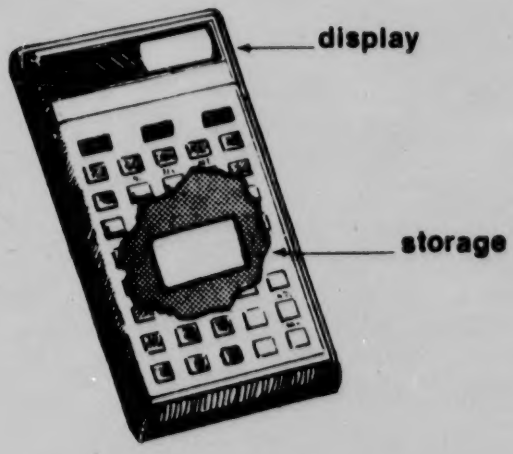


Step 4 – Press [=]

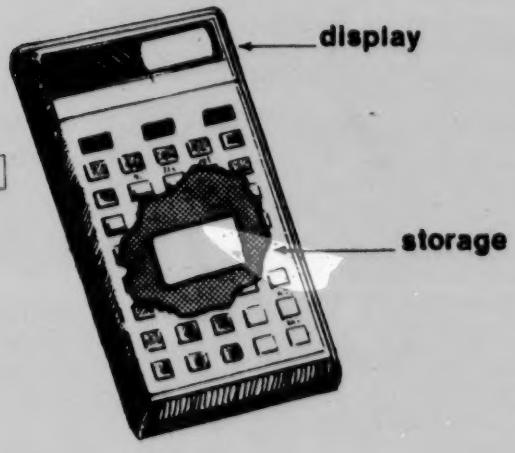


b. 13 - 6 = ?

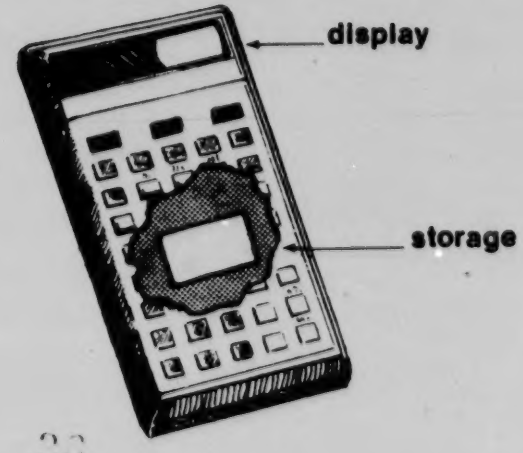
Step 1 – Enter 13



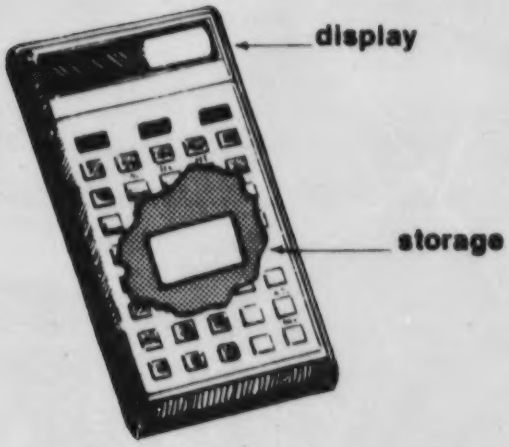
Step 2 – Press [-]



Step 3 – Enter 6



Step 4 – Press [=]



6. Complete the following simulation tables.

a. 5 + 8 = ?

ENTRY	DISPLAY	STORAGE

b. 25 - 10 = ?

ENTRY	DISPLAY	STORAGE

c. 12 + 45 - 10 + 3 = ?

ENTRY	DISPLAY	STORAGE	MEMORY

7. Perform the following addition and subtraction problems. *Do not clear* unless necessary. (When is it necessary?)

   a. 7 + 2 =

   b. 29 - 13 =

   c. 596 + 104 =

   d. 3059 - 1137 =

   e. 2103 + 471 + 6723 =

   f. 52,115 + 628,123 =

   g. 469 + 382 - 756 =

   h. -782 + 386 + 1502 =

   i. 46 + 831 - 643 - 186 =

   j. 654 + 937 - 386 + 27 - 1232 =

k. -372 - 678 + 1512 =

l. -13,572 + 5,782 + 678,316 =

m. 43.75 + 61.78 + 14.20 =

n. 43.030 + 0.125 + 1.370 =

o. 156.20 + 13.00 + 20.67 =

p. -17.2 + 4.3 + 21.73 =

q. 39,412 - 14,573.25 =

r. 0.00312 + 1.0157 - 0.00235 =

s. -52 - .003 + 152.75 =

t. 537.213 + 659.4 + 437.9 =

8. Check the answers to problems 7(h) – 7(p) by changing the order of the terms and redoing the problems.

9. In a series circuit in electricity, the total resistance is equal to the sum of the individual resistances.

$$R = R_1 + R_2 + \ldots$$

If R_1 = 80 ohms, R_2 = 80 ohms, R_3 = 80 ohms, R_4 = 350 ohms, and R_5 = 230 ohms, then the total resistance is

_____ ohms.

10. The capacitance of condensers joined in parallel equals the sum of the capacitances of the separate condensers.

$$C = C_1 + C_2 + C_3 + \ldots$$

If C_1 = 3, C_2 = 0.2, C_3 = 7, C_4 = 2.5, C_5 = 5.2, C_6 = 0.7, and C_7 = 6.5, then C =

_____ microfarads.

11. In preparing a balance sheet in accounting, the fundamental equation is:

Assets = Liabilities + Owner's Equity

What should the assets be of a company where the liabilities are $1135; $825; $43.25; $16,532.98; $14,737.15; $95.86; $127.89; and $7,380.79 and the Owner's Equity is $52,000.

_____

12. Another way of writing the above equation is:

Owner's Equity = Assets - Liabilities

James Reilly, owner of the Jim Dandy Plastics Company, has assets of $529,624 and liabilities of $498,700. What is his equity in the business?

_____

13. A salesman on the road for 3 days had the following expenses. Add them up to get his total on-the-road expenses for each day. Check your figures by adding across and comparing the two grand totals.

	Tuesday	Wednesday	Thursday	Totals
Motel	$ 18.72	$ 16.48	$ 21.20	
Breakfast	2.25	2.70	1.95	
Lunch	3.85	4.00	2.97	
Dinner	7.72	8.39	6.20	
Entertainment		18.50		
Mileage	20.25	14.25	9.45	
Tolls	1.50		.35	
Totals				

14. The Johnson family had an after-tax income of $1215 a month. Their fixed expenses were:

Rent	$285
Utilities; telephone	45
Transportation to work	39
Othe expenses related to job	75
Life insurance	32
Car insurance	28
Hospitalization	14

Figure out the amount available to the family for all their other needs (food, home maintenance and repairs, clothing, other expenses, and savings).

15. Check the following paychecks. Are any of them wrong?

		Deductions					
	Gross Pay	Federal Income Tax	State Income Tax	Unempl. Ins.	Hospital-ization	Union	Net Pay
R. Orlando	$124.79	$17.60	$1.05	$.62	$ .95	$1.88	$102.69
T. Lewis	148.02	13.80	.75	.74	2.00	2.22	127.51
H. Coleman	148.02	22.90	.95	.74	1.50	2.22	119.71
G. Stein	204.68	29.20	1.50	1.02	1.50	3.08	168.38
L. Garcia	189.54	27.20	2.05	.95	.95	2.85	165.54

16. Using the data below, find Susan Wood's checkbook balance as of March 15. If her bank statement showed a balance of $256.07, what was the difference (amount in outstanding checks)?

Balance, March 1, 19___ $ 279.17

Check No.	Date		
100	3/1	Jersey Central P & L	$ 22.01
101	3/1	Lou Grabber (rent)	$ 185.00
102	3/5	Shop Rite (groceries)	$ 60.00
	3/6	Deposit – paycheck	$ 188.04
103	3/8	Bamberger's (toaster)	$ 19.49
104	3/10	Cash	$ 40.00
105	3/10	First National Bank (loan)	$ 63.37
	3/13	Deposit – paycheck	$ 188.04
106	3/14	Redbook Magazine	$ 9.00
107	3/15	Sears (charge acct.)	$ 55.39
108	3/15	New York Life (Insurance)	$ 40.34

Balance, March 15 _____

Difference _____

17. Using the data below, find Ms. Wood's balance as of April 5.

		Balance, March 20, 19____	**$ 160.65**
Check No.	**Date**		
109	3/20	City Water Co.	$ 28.21
110	3/21	Daily Home News	$ 6.54
	3/22	Deposit – paycheck, less cash	$ 138.04
111	3/22	Red Cross (flood relief)	$ 15.00
112	3/25	Stirling hardware (gutters for porch)	$ 39.31
	3/27	Deposit – paycheck	$ 188.04
113	3/30	Cash	$ 40.00
114	3/30	New Jersey Bell (telephone)	$ 16.88
115	4/1	Lou Grabber (rent)	$ 185.00
116	4/1	Shop Rite	$ 60.00
	4/3	Deposit – paycheck	$ 188.04
117	4/5	Chrysler Corp.	$ 86.47

18. On the next pages you will find two copies of a form that the Last National Bank furnishes its depositors to help them reconcile their bank statements with their checkbooks. Using the figures below, fill in the first form as it directs, and reconcile this account for January.

January, 19____		**Outstanding checks**	
Statement balance	$1559.38	**Check No.**	**Amount**
Deposits not credited	$ 100.00	340	$ 109.00
Present checkbook bal.	$ 15.43	3[illegible]9	$ 21.65
Service charge	$ 1.90	351	$ 260.20
		357	$1000.00
		358	$ 10.00
		359	$ 245.00

19. Using the figures below and the second form, reconcile this account for February.

February, 19____		**Outstanding checks**	
Statement balance	$ 25.58	**Check No.**	**Amount**
Deposits not credited	$ 335.82	363	$ 1.31
Present checkbook bal.	$ 340.54	371	$ 3.00
Service charge	$ 1.45	372	$ 8.00
		373	$10.00

The Last National Bank of New Jersey

Ayes Crossing, New Jersey 08000

Month _____ 19____

This form is provided to help you balance your bank statement.

Checks Outstanding
Not Charged to Account

Check No.	Amount	
TOTAL		

Bank Balance shown on statement _____

Add (+)
Deposits not charged in this statement (if any) _____

Sub-Total _____

Subtract (-)

Checks outstanding _____

Checkbook Balance _____

Subtract (-)

Service charge _____

Present Checkbook Balance _____

Balance _____

These amounts should agree

The Last National Bank of New Jersey

Ayes Crossing, New Jersey 08000

Month _____ 19____

This form is provided to help you balance your bank statement.

Checks Outstanding Not Charged to Account

Check No.	Amount	
TOTAL		

Bank Balance shown on statement _____

Add (+)
Deposits not charged in this statement (if any) _____

Sub-Total _____

Subtract (-)
Checks outstanding _____

Checkbook Balance _____

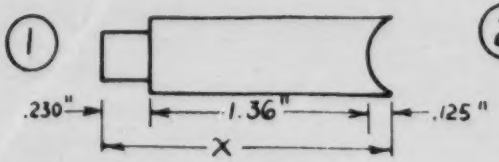
Subtract (-)
Service charge _____

Present Checkbook Balance _____

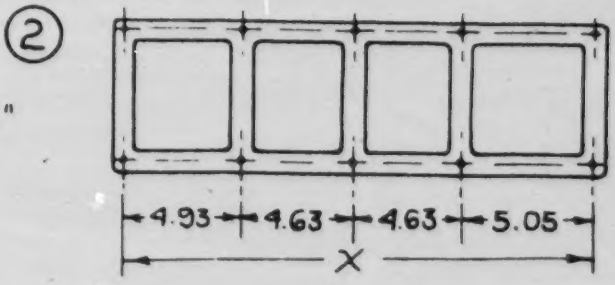
Balance _____

These amounts should agree

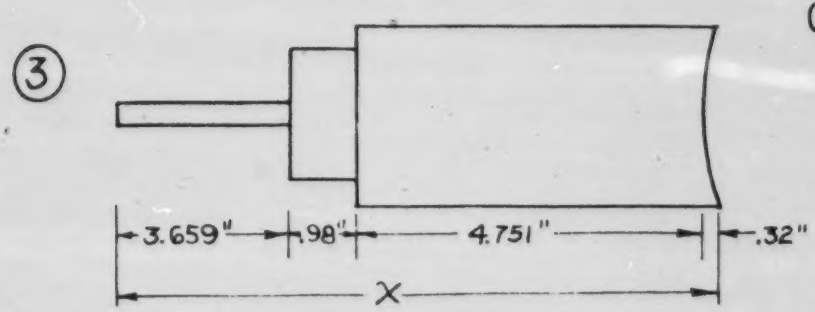
20. Find the overall length of each product shown here.



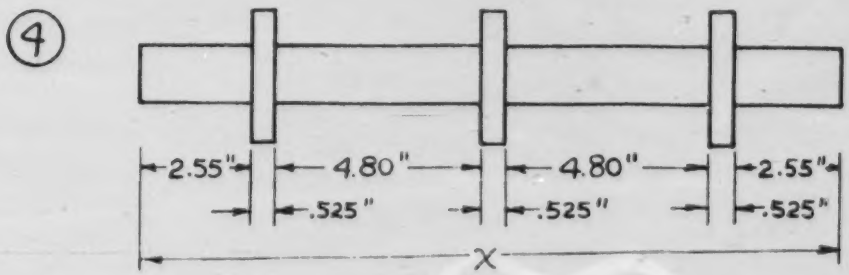
Carbon Brush



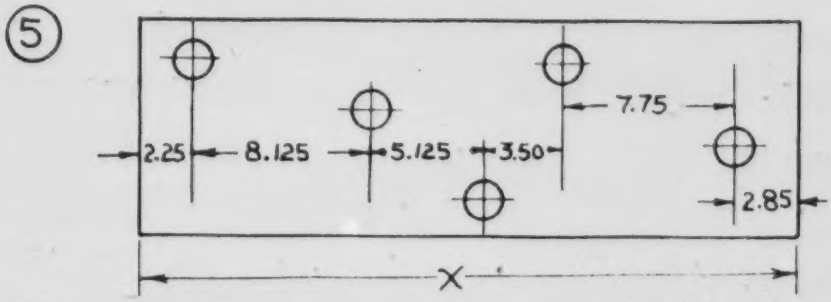
Gasket



Carbon Alloy Brush



Roller



Electrical Panel Board

21. You are a front-end manager in your nearby supermarket. You must compare the sales for two different weeks for each department. If there is an increase the second week, write the amount in the Increase column; if there is a decrease, write the amount in the Decrease column.

    Total up all columns and check your own figures.

Department	Week 1	Week 2	Increase	Decrease
Grocery	$15,396.48	$18,795.31		
Produce	$ 3,506.07	$ 4,136.98		
Meat	$18,107.86	$17,746.08		
Dairy	$ 1.962.88	$ 2,355.10		
Delicatessen	$ 738.25	$ 961.23		
Fish	$ 361.49	$ 302.98		
Drugs	$ 572.21	$ 599.83		
Bakery	$ 600.47	$ 492.21		
TOTALS				

II – FUNDAMENTAL OPERATIONS

Lesson 2 — **Multiplication and Division**

Objectives:

1. You should be able to use your calculator to perform multiplication and division problems.
2. You should be able to explain the commutative property of multiplication.
3. You should be able to correct an incorrectly pressed function key (multiplication or division).
4. You should know whether or not it is necessary to clear the registers before starting a new problem.
5. You should be able to complete simulation tables of multiplication and division problems.

Information:

Like addition and subtraction, multiplication and division are binary operations (see page 13). Therefore, we will need two registers to work with.

The operation of multiplication is commutative. Just as in addition, the order in which we multiply will not change the answer. ($a \cdot b = b \cdot a$)

Examples: $2 \cdot 3 = 3 \cdot 2 = 6$

$5 \cdot 3 \cdot 2 = 3 \cdot 2 \cdot 5 = 2 \cdot 5 \cdot 3 = 30$

Like subtraction, division is not commutative. If we interchange the numbers in a division problem, we will not get the same answer.
($a \div b \neq b \div a$)

Examples: $2 \div 6 = \frac{1}{3}$

$6 \div 2 = 3$

therefore, $2 \div 6 \neq 6 \div 2$

In a chain of multiplication and division, however, we can change the order any way that we wish. (Of course we have to keep the X and ÷ signs *with the numbers they apply to.)*

Example: $6 \cdot 3 \div 2 \cdot 5 \div 9 = 5$

$3 \div 2 \cdot 6 \cdot 5 \div 9 = 5$

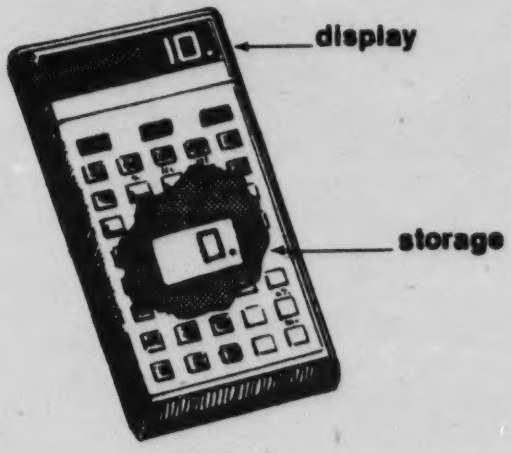
$6 \div 2 \div 9 \cdot 3 \cdot 5 = 4.9999995$ (=5)

and so on.

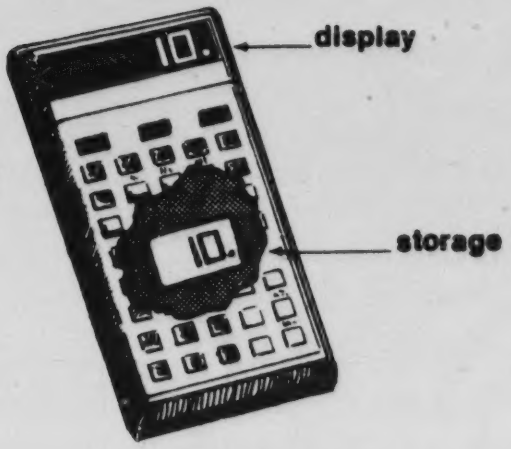
Example: Multiplication

$10 \cdot 9 = ?$

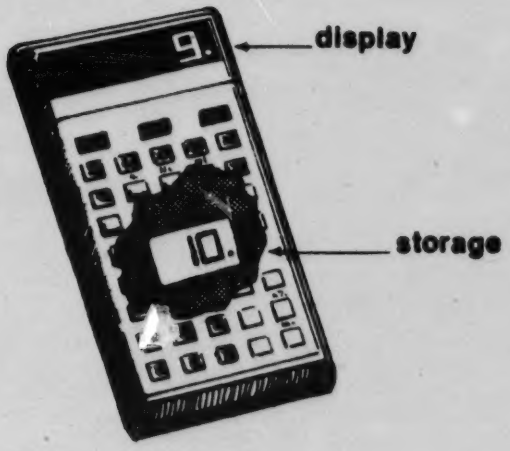
Step 1 – Enter 10



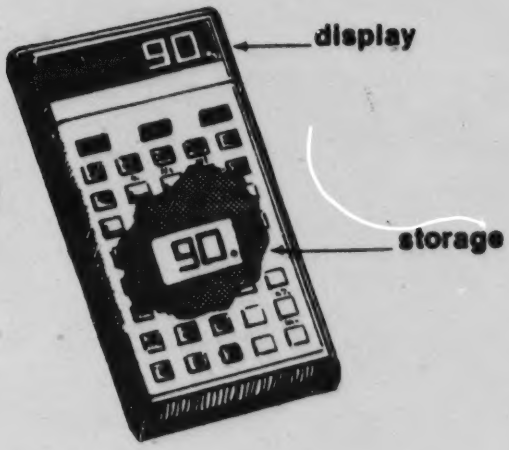
Step 2 – Press [X]



Step 3 – Enter 9



Step 4 – Press [=]



By simulation table: $9 \cdot 10 = ?$

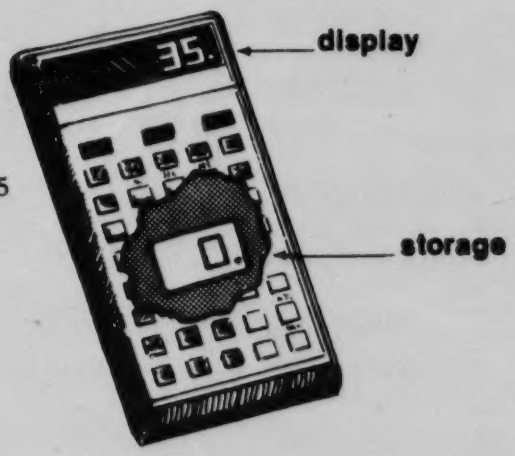
ENTRY	DISPLAY	STORAGE
9	9.	0.
[X]	9.	9.
10	10.	9.
[=]	90.	90. (9.) (10.)*

* Different calculators place different numbers in the storage register when the [=] key is pressed. You can find out what your own calculator does by pressing the [x/y] or [x↔y] key, if your calculator has one.

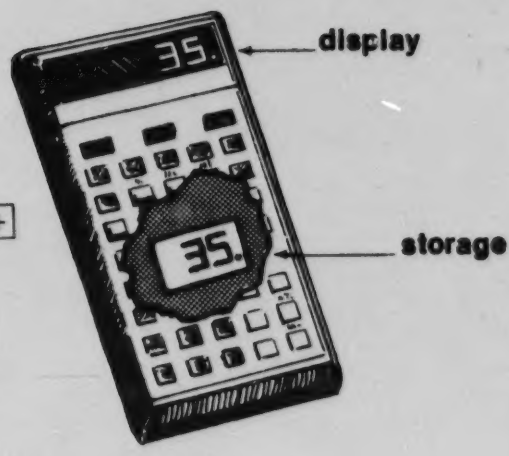
Example : Division

$35 \div 5 = ?$

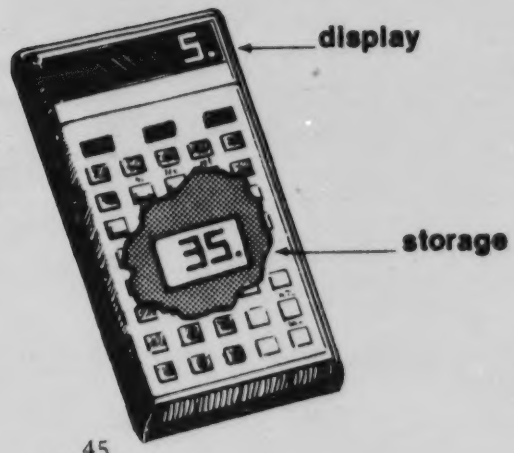
Step 1 – Enter 35



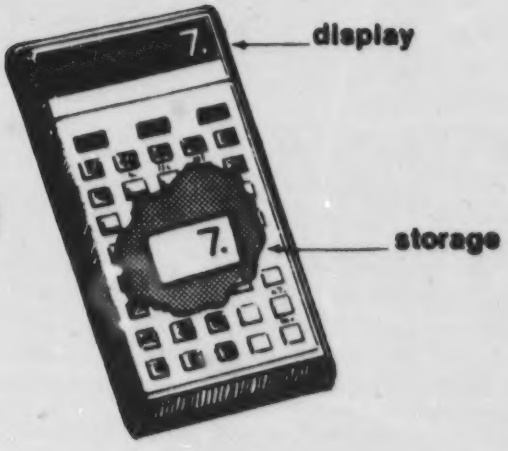
Step 2 – Press ÷



Step 3 – Enter 5



Step 4 – Press [=]



By simulation table: $35 \div 5 = ?$

ENTRY	DISPLAY	STORAGE
35	35.	0.
[÷]	35.	35.
5	5.	35.
[=]	7.	7.

Example: Chain multiplication and division

$6 \cdot 3 \div 9 \cdot 4 \cdot 3 \div 12 = ?$

ENTRY	DISPLAY	STORAGE
6	6.	0.
[×]	6.	6.
3	3.	6.
[÷]	18.	18.
9	9.	18.

(continued on next page)

ENTRY	DISPLAY	STORAGE
[X]	2.	2.
4	4.	2.
[X]	8.	8.
3	3.	8.
[÷]	24.	24.
12	12.	24.
[=]	2.	2.

Notice that pressing any function key gives us *the answer up to that point.* Also, this answer is reached by *operating on* (X or ÷) the number in the display and the number in the storage.

If we multiply a number by 1 or divide a number by 1, the result will be the number itself. ($a \cdot 1 = a$ and $a \div 1 = a$). This is important to our understanding of the process by which we can correct an incorrectly pressed [X] or [÷] key.

Example: $32 \div 4 = ?$

	ENTRY	DISPLAY	STORAGE
	32	32.	0.
Mistake →	[X]	32.	32.
Correction	1	1.	32.
	[÷]	32.	32.
	4	4.	32.
	[=]	8.	8.

or 32 [X] 1 [÷] 4 [=] 8

Notice that when we entered the 1 and then pressed the [÷] key the result of the multiplication was placed in both the display and storage registers.

Example: $14 \cdot 5 = ?$

	ENTRY	DISPLAY	STORAGE
	14	14.	0.
Mistake →	[÷]	14.	14.
	1	1.	14.
Correction →	[X]	14.	14.
	5	5.	14.
	[=]	70.	70.

or 14 [÷] 1 [X] 5 [=] 70

Most calculators will automatically insert the 1 entry for us if we press the [X] key immediately after the [÷] key or vice versa. Therefore, it is not necessary for the user to make the entry of 1 (as was done in the two preceding examples).

Example: $4 \cdot 5 \div 10 = ?$

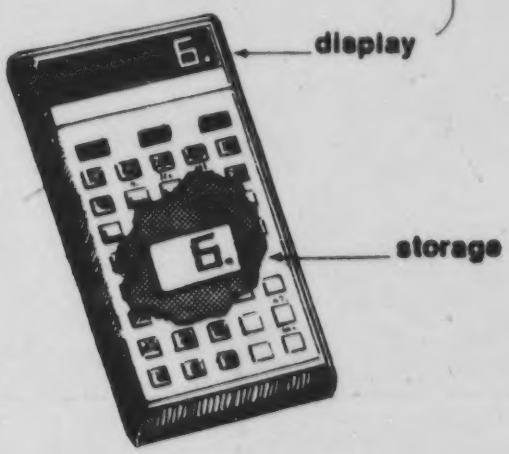
	ENTRY	DISPLAY	STORAGE
	4	4.	0.
Mistake →	[÷]	4.	4
	[X]	4.	4.
Entry of 1 not necessary →	5	5.	4.
	[X]	20.	20.
Mistake →	[÷]	20.	20.
	10	10.	20.
	[=]	2.	2.

or 4 [÷] [X] 5 [X] [÷] 10 [=] 2

Therefore, we can simply state that the [X] and [÷] keys will cancel each other if pressed in sequence.

Another principle that is very similar to what we learned about addition and subtraction is the fact that we do not have to clear the registers in order to start a new problem – *as long as the new problem starts with a data key and not a function key.* Also, the previous problem must have been ended with the use of the [=] key.

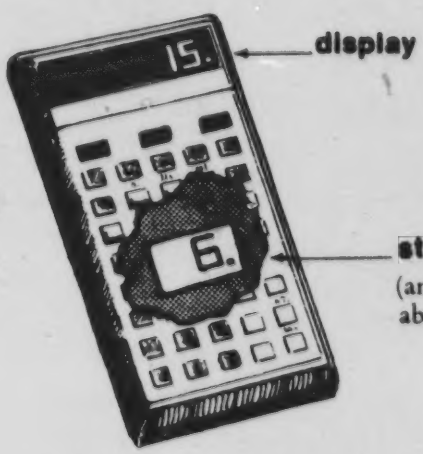
Example: 3 · 2 = 6 Answer



If we decide to start another problem, we can proceed without clearing the registers.

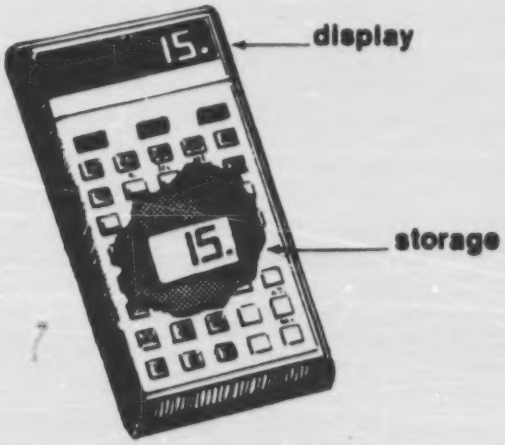
Example: 15 · 3 = ?

Step 1 – Enter 15



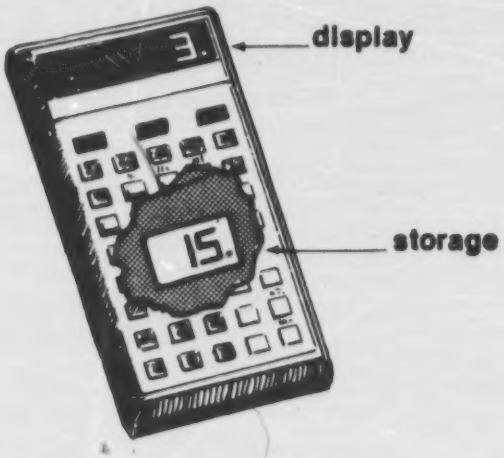
Notice that the 15 replaced the 6 in the display by destructive read-in.

Step 2 – Press [X]

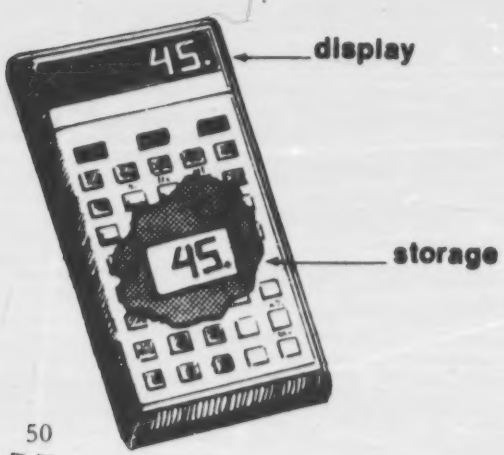


Now the 15 is duplicated in the storage register, destroying the 6 there also.

Step 3 – Enter 3



Step 4 – Press [=]



Questions:

1. Which of the following operations are commutative?

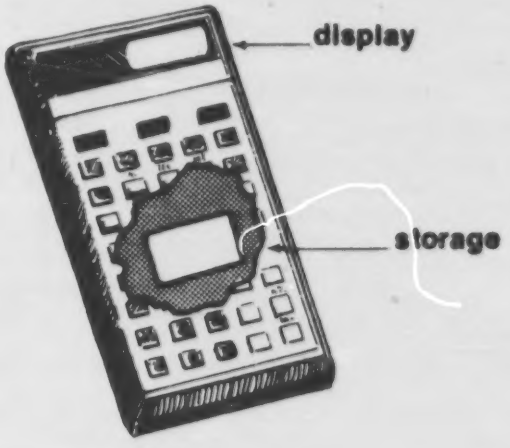
   a. addition
   b. subtraction
   c. multiplication
   d. division

2. Press the following sequence of keys and see if you receive the indicated answer. If you do not, then check your owner's manual to find out how to correct an incorrectly pressed [X] or [÷] key. Another possibility is to enter a 1 and continue with the correct function.

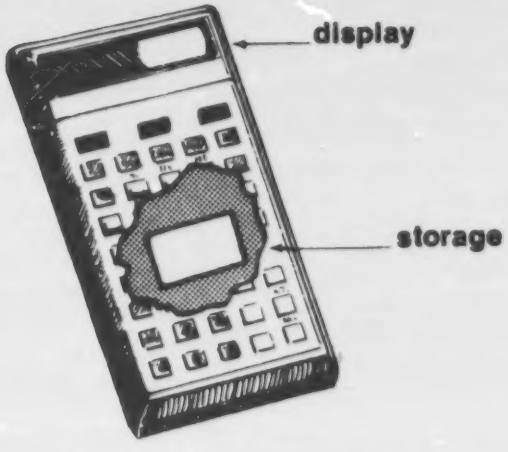
   a. [6] [÷] [X] [2] [=] 12 — answers
   b. [8] [X] [÷] [2] [=] 4 — answers

3. Fill in the display register and the storage register for each of the following:

   a. 15 · 3 = ?

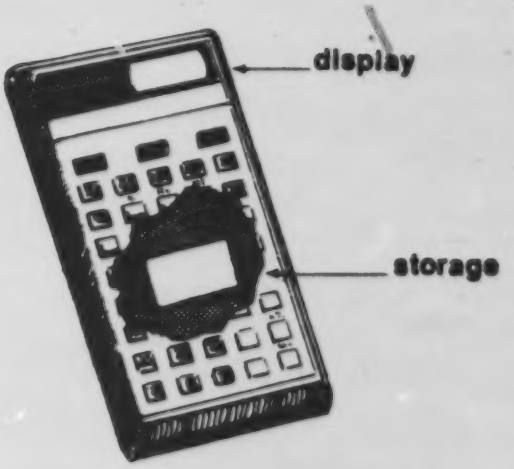


Step 1 – Enter 15

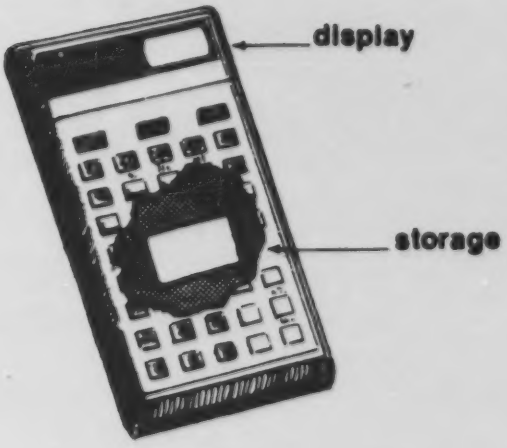


Step 2 – Press [X]

Step 3 – Enter 3

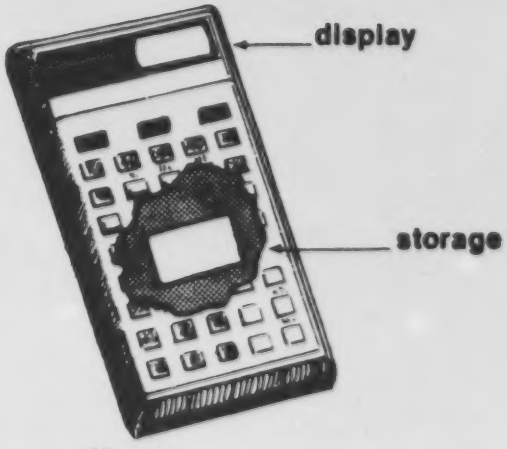


Step 4 – Press [=]



b. $100 \div 25 = ?$

Step 1 – Enter 100



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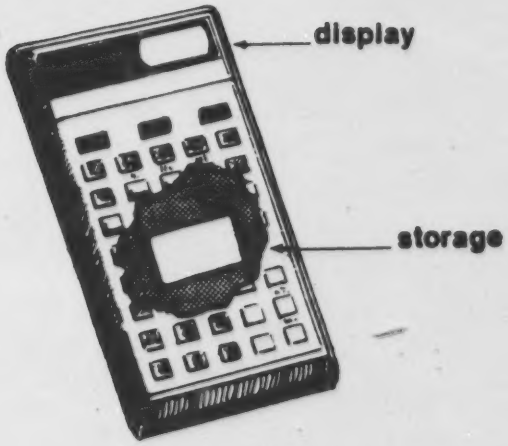
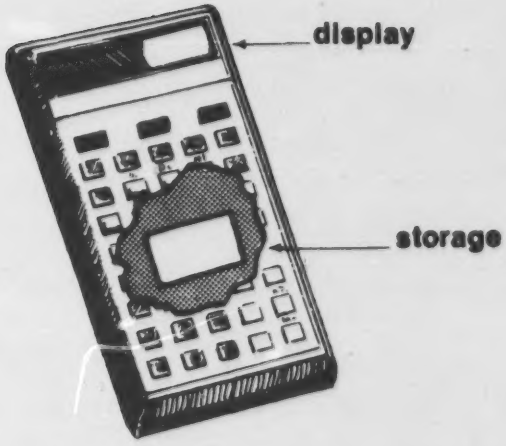
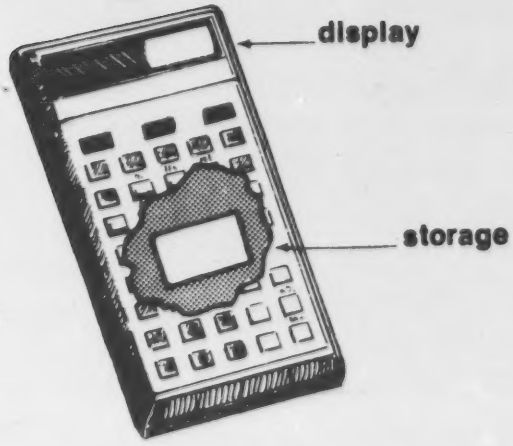
Step 2 – Press [÷]



Step 3 – Enter 25



Step 4 – Press [=]



4. Complete the following simulation tables.

a. $7 \cdot 9 = ?$

ENTRY	DISPLAY	STORAGE

b. $90 \div 6 = ?$

ENTRY	DISPLAY	STORAGE

c. $5 \cdot 3 \cdot 2 \div 15 = ?$

ENTRY	DISPLAY	STORAGE

5. Perform the following multiplication and division problems. *Do not clear unless necessary.* (When is it necessary?)

   a. $15 \cdot 2 =$

   b. $40 \div 5 =$

   c. $497 \cdot 23 =$

   d. $3051 \cdot 19 =$

   e. $51{,}968 \div 56 =$

   f. $19 \cdot 32 \cdot 7 =$

   g. $85 \div 17 =$

   h. $218{,}400 \div 65 \div 96 =$

   i. $58{,}968 \div 78 \cdot 2 \div 54 =$

   j. $1489 \cdot 3 \cdot 11 \cdot 16 \div 132 =$

   k. $5766 \div 93 \cdot 4 \div 16 =$

   l. $79 \cdot 13 \cdot 32 \div 16 =$

   m. $14.7 \cdot 18 \div 5 =$

   n. $13.75 \cdot 25 \div 0.25 =$

   o. $17.6 \div 0.002 \div 220 =$

   p. $0.033 \cdot 0.051 \div 0.003 =$

   q. $45.3 \div 0.03 \cdot 5 \div 15.1 =$

   r. $0.007 \div 0.001 \cdot 7.39 =$

   s. $49 \div 7 \div 0 =$

   t. $65.321 \cdot 5.8 \cdot 0 =$

6. In basic electricity we have the formula

$$W = E\,I$$

(watts = volts · amps)

(Where letters are placed next to each other in formulas, as above, multiplication is to be performed.)

If E = 120 volts (house current), and I = .8 amps (light bulb), then W = _____ watts.

7. According to Ohm's law, amperes of current (I) equals electromotive force in volts (E) divided by resistance in ohms (R).

$$I = \frac{E}{R}$$

If E = 12 volts and R = 8 ohms, then I = _____ amps.

8. If $I = \frac{E}{R}$ then $E = IR$, and $R = \frac{E}{I}$

a. Find E (in volts) if I = 0.004 amps and R = 50,000 ohms. _____ volts

b. Find R (in ohms) if E = 120 volts and I = 8 amps. _____ ohms

9. The resistance of a wire can be found by using the formula

$$R = \frac{K \cdot L}{C.M.}$$

where: R = resistance
K = resistance in ohms of one mil-foot of the kind of wire used
L = the length of the wire in feet
C.M. = the circular cross-section measure of the wire in mils

If L = 150 ft., K = 17, and C.M. = 1020, then R = _____ ohms.

10. To convert a temperature from Celsius to Fahrenheit, the formula is

$$°F = \frac{9}{5} \cdot °C + 32$$

Find the temperature in Fahrenheit when

a. °C = 20° °F = _____

b. °C = 37° °F = _____

c. °C = 100° °F = _____

d. °C = 0° °F = _____

11. In working with levers, the formula

$$W_A = \frac{W_B \cdot d_B}{d_A}$$

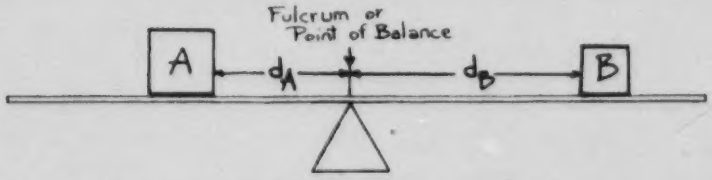
can be used to find an unknown weight (W_A) so that a balanced system (state of equilibrium) is produced.

In the formula, W_A = the weight needed at point A

W_B = the weight at point B

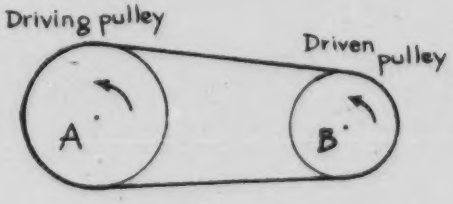
d_B = the distance B is from the fulcrum

d_A = the distance A is from the fulcrum



If W_B = 20 kg, d_B = 168 cm, and d_A = 105 cm, then W_A = _____ kg.

12.



In working with pulleys, the formula

$$rpm_B = \frac{rpm_A \, d_A}{d_B}$$

can be utilized to find the revolutions per minute of the driven pulley (rpm_B) when given the revolutions per minute of the driving pulley (rpm_A), the diameter of the driving pulley (d_A) and the diameter of the driven pulley (d_B).

If rpm_A = 68 rpm, d_A = 35 cm, and d_B = 20 cm, then rpm_B = _____ rpm.

If rpm_A = 3000 rpm, d_A = 17 cm, and d_B = 8 cm, then rpm_B _____ rpm.

13. If one gear is driving another, then the formula

$$rpm_B = \frac{T_A \cdot rpm_A}{T_B}$$

can be used to find the revolutions per minute of the driven gear.

rpm_B = revolutions per minute of the driven gear

T_A = the number of teeth in the driving gear

rpm_A = the revolutions per minute of the driving gear

T_B = the number of teeth in the driven gear

If T_A = 48, T_B = 96, and rpm_A = 500 rpm, then rpm_B = _____ rpm.

14. In a certain compound gear train the formula

$$rpm_D = \frac{rpm_A \cdot T_A \cdot T_C}{T_B \cdot T_D}$$

yields the rpm of the final driven gear in the train. The rpm_A = the revolutions per minute of the first driving gear, T_A and T_C are the number of teeth in the driving gears, and T_B and T_D are the number of teeth in the driven gears. If rpm_A = 400 rpm, T_A = 30, T_C = 72, T_B = 40, and T_D = 24, then rpm_D =

_____ rpm.

15. In air conditioning and heating, the air capacity of a 6-inch duct (C) equals the air velocity (V) times the cross-sectional area (A).

$$C = VA$$

If V = 1,500 fpm (feet per minute) and A = .196 sq. ft., then C = _____ cfm (cubic feet per minute).

16. At constant air volume (cubic feet per minute) and fan speed (feet per minute) the static pressure (P) in a duct system varies in direct proportion to the air density.

$$\frac{P_1}{P_2} = \frac{d_1}{d_2}$$

Therefore,

$$P_2 = P_1 \cdot \frac{d_2}{d_1}$$

where:

P_1 = static pressure for initial condition, measured in inches of water

P_2 = static pressure for final condition

d_1 = density for initial air temperature

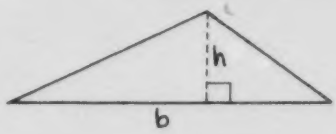
d_2 = density for final air temperature

If P_1 = 0.25 in. of water, d_1 = 0.0640 lb. per cu. ft., and d_2 = 0.0763 lb. per cu. ft., then P_2 =

_____ inches of water. (Drop the last few decimals.)

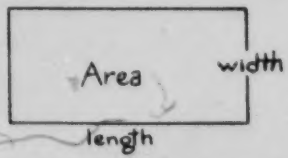
17. The area of any triangle is equal to one-half the product of the base and the height.

$$A = \frac{1}{2} bh$$



If b = 8.3 cm and h = 23.5 cm, then A = _____ cm^2.

18. The area of a rectangle is equal to the length times the width.

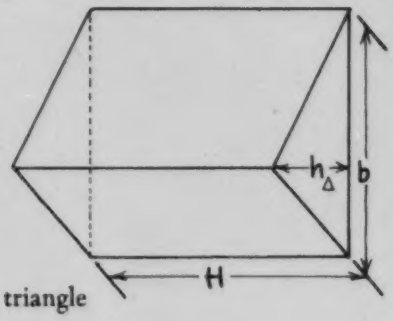


Complete the following table. Write in the proper unit each time.

	Length	Width	Area
a)	4,500 ft.	295 ft.	
b)	87.35 m	143.2 m	
c)	9,821.6 ft.	67.5 ft.	
d)	56.27 mm	8.59 mm	
e)	63.7 cm	95.81 cm	

19. The volume of a right triangular prism is given by the formula

$$V = \frac{1}{2} bh_{\Delta} H$$



where: V = the volume of the prism

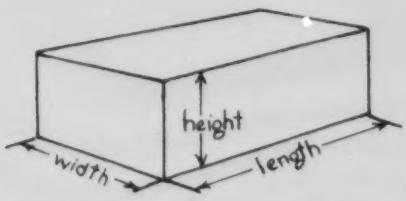
b = the length of the base of the triangle

h_{Δ} = the height of the triangle

H = the height of the prism

If b = 47.8 cm, h_{Δ} = 29 cm, and H = 98.4 cm, then V = _____ cm^3.

20.



The volume of a box is equal to the length times the width times the height.

$$V = lwh$$

a. A fish tank is 100 cm long, 60 cm wide, and 80 cm high. The volume is _____ cm^3.

b. A room is 2.75 metres wide, 3.66 metres long, and 2.44 metres high (9 ft. by 12 ft. by 8 ft.). The volume is _____ cubic metres.

c. The floor of a new house is a concrete slab 7.62 m by 9.05 m by 0.40 m thick. The volume is _____ m^3.

21. In the study of probability we can use the formula

$$n = m \cdot p \cdot r \cdot s \cdot t$$

to find the number of different 5-digit zip codes possible. In the formula (based on the fundamental principle of counting), n = the total number of possibilities; m, p, r, s, and t = the number of possibilities for each digit of the zip code. If m = 10, p = 10, r = 10, s = 10, and t = 10, then n = _____

If no repetitions of digits is permitted, then the number of possibilities is reduced, and m = 10, p = 9, r = 8, s = 7, and t = 6. For this case, find the number of possibilities: n = _____

22. What will be the weight of a carton of 24 cans of fruit if each can weighs 1.4 pounds? _____ lb.

23. What is the cost of four folding chairs if each one costs $7.98? _____

24. What should be the cost of 4 rolls at the deli counter if rolls sell for $1.10 a dozen? _____

25. How many cubic feet of concrete are needed for a driveway 42 feet long by 12 feet wide by .75 feet thick? _____ cu. ft.

How many cubic yards is that? _____ cu. yd.

26. How many cubic inches in a cubic yard? _____ cu. in.

27. If bricks are laid with half-inch mortar, 19 of them will fill a cubic foot. How many bricks will it take to build a wall 24 feet long, 8 feet high, and 1 foot thick? _____ bricks

28. In mechanics, the formula for work done is

$$W = F \cdot D$$

where W = work, measured in foot-pounds
F = force, in pounds
D = distance, in feet

How much work is done in lifting a 1400-pound piano a distance of 3 *inches* off the ground?

_____ ft.-lb.

29. Find the average of these six test marks: 85, 92, 67, 75, 100, 80.

_____

30. You work for Shop Rite, and it is time to inventory your stock. Your job is to find out the number of cans of each product in one section of the storeroom. You count the cartons and write down all the information you gather, in the table shown below. Now complete the table for each item and get the total of all the items.

Item	Number of Cartons	No. Cans Per Carton	Total No. of Cans
Niblet Corn 16 oz. Green Giant	8	24	
Cut Wax Beans 16 oz. Green Giant	14	24	
Cream Corn 8 oz. Green Giant	6	48	
Cream Corn 16 oz. Green Giant	32	24	
Green Peas 8 oz. Green Giant	18	48	
Green Peas 16 oz. Green Giant	12	24	
Whole Potatoes 8 oz. Green Giant	10	48	
Whole Potatoes 16 oz. Green Giant	8	24	
Sliced Potatoes 16 oz. Green Giant	11	24	
Tomato Sauce 8 oz. Del Monte	13	72	
Tomato Sauce w/onions 8 oz. Del Monte	19	72	
Tomato Puree 16 oz. Del Monte	16	24	
Tomato Puree 28 oz. Progresso	24	12	
TOTAL			

31. You are buying salad fixings for the DeLuxe Diner. Figure out your total bill.

8 heads iceberg lettuce	@ .49	_____
2 heads romaine	@ .39	_____
4 pounds chicory	@ .35/lb.	_____
15 cucumbers at	@ .17	_____
3 stalks celery	@ .49	_____
5 pounds tomatoes	@ .39/lb.	_____
TOTAL		_____

32. Here's how to find the number of miles per gallon a car gets. Start with a full tank and note the mileage on the car's odometer. Then the next time you need gas, note how many gallons it takes to fill the tank. Find out how many miles you have gone by subtracting the previous odometer reading from the new reading. Then divide:

$$\text{Miles per gallon} = \frac{\text{No. of miles traveled}}{\text{No. of gallons used}}$$

Find the number of miles per gallon in the following cases. (Give your answer to the nearest *whole number.)*

Odometer Readings	No. of Gals. Used	Miles Per Gallon
31652 – 31400	11.0	_____
22515 – 22326	9.3	_____
1496 – 1047	15.5	_____
49854 – 49629	18.5	_____
19121 – 18919	7.2	_____

33. The high temperatures for the week of July 4–10 were (in degree Celsius): 29°, 32°, 28°, 24°, 25°, 30°, and 33°. What was the average high that week?

_____ °C

34. Ralph Roller, a painter, figured on covering about 350 square feet of surface with each gallon of paint. His job called for about 195 running feet of wall, 12 feet high. How many gallons of paint did he need to buy?

_____ gal.

35. What pay would Peter receive on his after-school work for this week? He earns $2.65 per hour.

Monday	4 hours	_____
Tuesday	3.5 "	_____
Wednesday	2.75 "	_____
Friday	5 "	_____
	Total	_____

36. Chunkaroos cereal comes in three sizes:

8 oz. box for $.63
12 oz. box for $.79
18 oz. box for $1.29

Which is the best buy? _____

37. Overtime is paid for at 1.5 times the regular rate. Figure the overtime pay rate for the following people:

	Regular Hourly Rate	Overtime Rate
R. Colson	$ 2.86	_____
T. Martinez	3.10	_____
K. O'Neill	4.75	_____
B. Gomberg	2.95	_____
B. Korsak	4.56	_____

38. The dimensions of a room are 12.5′ X 18′. How many floor tiles will be needed if each tile is 1 square foot?

_____ tiles

39. "Thirty-penny" nails run about 24 to the pound. A carpenter estimates that he'll need about 200 of that size for a job. How many pounds should he get?

_____ lb.

40. A wall face using a 3/8″ mortar joint takes 6 2/3 (6.67) brick per square foot. A mason estimates that a brick-veneer house he is working on has 550 square feet to be covered with brick. To the nearest 100, how many bricks should he order?

_____ bricks

41. In buying insulation, it is important to know about its "R" value – its resistance to the flow of heat. The higher the R, the better the insulation. Find the R value of

a. 4 inches of mineral-wool batt @ R of 3.80 per inch _____

b. 6 " " " " " " " " " " " _____

c. 4 inches of loose-fill mineral wool @ R of 3.30 per inch. _____

d. 6 " " " " " " " " " " " " _____

42. To find the capacity of an air conditioner required for any given place, the formula used is

$$C = \frac{W \cdot H \cdot I \cdot L \cdot E}{60}$$

Where C = capacity in BTU's
W = width of area to be cooled, in feet
H = height of area to be cooled, in feet
I = insulation factor (10 for well-insulated, up to 18 for poorly insulated or with a lot of windows)
L = length of area, in feet
E = exposure (north = 16, east = 17, south = 18, west = 20)

You wish to cool a room 13' X 15' that is 8 feet high. It has several windows, so you decide to use 18 for the insulation factor. Its longest outside wall faces east. Find the capacity of air conditioner you require.

_____ BTU

43. To find the energy-efficiency rating (E.E.R.) of an air conditioner, divide the unit's output capacity by its input. The higher the E.E.R., the more efficient the air conditioner.

$$\text{E.E.R.} = \frac{\text{Output in BTU's}}{\text{Input in watts}}$$

Find the EER of:

a. Brand X with an output of 8000 BTU's if it requires 1350 watts to operate. (Keep one decimal place.)

_____

b. Brand Y with the same output but which requires 860 watts to operate.

_____

Which brand would you buy? _____

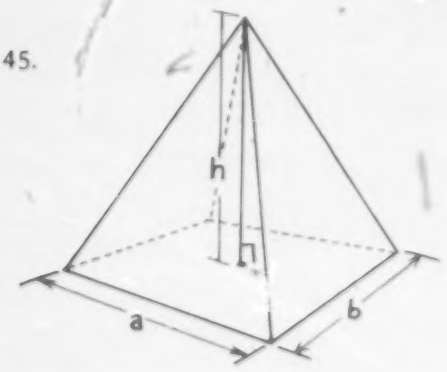
44. John Jennings drove his car 12,604 miles last year. He kept a running account of his expenses, as follows:

Gasoline	$ 397.36
Oil changes and oil added	26.50
Insurance	289.00
Estimated depreciation	950.00
Servicing and repairs	245.00
License plates	24.00
Miscellaneous	42.16

How much did his car cost him per mile? (to the nearest cent)

_____

45.



The volume of a pyramid can be found by using the formula

$$V = \frac{1}{3}abh$$

where a and b are the dimensions of the base and h is the height of the pyramid. If a = 38.62 mm, b = 46.0 mm, and h = 73.12 mm, then V =
(Round off to the nearest tenth) _____ mm^3

46. The cutting speed on a lathe is the speed with which the piece being machined revolves against the cutting tool. It is measured in feet traveled per minute.

$$C.S. = \frac{rpm \times d}{4}$$

Where rpm = speed of piece in revolutions per minute
d = diameter of piece in inches

Find the cutting speed of a piece of work 3.75" in diameter that is turning at 165 rpm (to the nearest whole number).

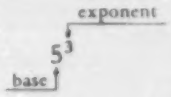
_____ ft/min

Lesson 3 — **Powers and Roots**

Objectives:
1. You should be able to raise a number to a given power.
2. You should be able to find the square root of a number.
3. You should be able to find higher powers and roots.

Information: An **exponent** tells us how many times to use the base as a factor (a multiplier.) Another word for exponent is **power.**

Example:



Read as "5 to the third power" or "5 cubed".

$$5^3 = \underset{(1)}{5} \cdot \underset{(2)}{5} \cdot \underset{(3)}{5} = 125$$

Therefore, we can raise any base to the indicated power (exponent) by repeated multiplications.

Example: $6^8 = ?$

ENTRY	DISPLAY	STORAGE
6	6.	0.
[X]	6.	6.
6	6.	6.
[X]	36.	36.
6	6.	36.
[X]	216.	216.
6	6.	216.
[X]	1296.	1296.
6	6.	1296.
[X]	7776.	7776.
6	6.	7776.
[X]	46656.	46656.
6	6.	46656.
[X]	279936.	279936.
6	6.	279936.
[=]	1679616.	1679616.

6 used as a factor 8 times

The amount of work involved in the problem above was considerable. On many calculators we find the function key $\boxed{X^Y}$ or $\boxed{Y^X}$. Here is a simulation table illustrating the previous problem, but using this key.

Find $6^8 = ?$

	ENTRY	DISPLAY	STORAGE
Enter base first	6	6.	0.
	$\boxed{Y^X}$	6.	6.
Enter exponent	8	8.	6.
	$\boxed{=}$	1679616.	1679616.

Obviously this key can save the user considerable time when raising a number to a power.

The most frequently used power is 2. Many calculators have a specific key $\boxed{X^2}$ that squares a number in the display register. Considering that squaring a number is the same as using that number as a factor only two times ($x^2 = x \cdot x$) we can see that this key is certainly not as powerful as $\boxed{X^Y}$. However, for people who do not work much with higher powers – and that means most people – it is quite adequate.

Example: $37^2 = ?$

ENTRY	DISPLAY	STORAGE
37	37.	0.
$\boxed{X^2}$	1369.	0.

Note that the answer, 1369, appears only in the display register. (It will be transferred to the storage register if another function key is pressed.)

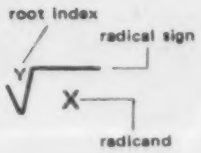
Study each of the following examples.

a. $3^2 = 9$ $\sqrt{9} = 3$

b. $5^3 = 125$ $\sqrt[3]{125} = 5$

c. $4^4 = 256$ $\sqrt[4]{256} = 4$

In the right column of the examples shown above we are finding a root of the given number.



The **root index** tells the type of root we are to find. Notice that the 2 was understood in example (a) above. This is true for square roots only. All others will be clearly indicated.

The **radical sign** is the special sign used when we are finding the root of a number.

The **radicand** is the number for which we are to find the indicated root.

On scientific calculators, the $\boxed{\sqrt[Y]{X}}$ or $\boxed{\sqrt[X]{Y}}$ key is used to find the root of a given number.

Example: Find $\sqrt[8]{1679616}$

	ENTRY	DISPLAY	STORAGE
Enter radicand first →	1679616	1679616.	0.
	$\boxed{\sqrt[Y]{X}}$	1679616	1679616.
Enter root index →	8	8.	1679616.
	$\boxed{=}$	6.	6.

As with the case of powers, the most frequently found root is the square root (root index = 2). Many calculators have a specific key $\boxed{\sqrt{x}}$ that finds the square root of a number in the display.

Example: F $\sqrt{625}$

ENTRY	DISPLAY	STORAGE
625	625.	0.
$\boxed{\sqrt{x}}$	25.	0.

To find the indicated root of a number without the key $\boxed{\sqrt[x]{y}}$ is much more difficult than raising to a power (repeated multiplications). We will not go into the methods of doing this here. It is because of this difficulty that the $\boxed{\sqrt[x]{y}}$ is a very desirable key for those whose work will involve roots higher than 2.

In working with powers and roots, the machine will only find the power or root of the *number in the display*. If any operation must be done before taking the power or the root, you must perform the operation, press the $\boxed{=}$ key, and then take the power or the root.

Example: $\sqrt{75 + 25} = ?$

ENTRY	DISPLAY	STORAGE
75	75.	0.
$\boxed{+}$	75.	75.
25	25.	75.
$\boxed{=}$	100.	100. (25.)
$\boxed{\sqrt{x}}$	10.	100. (0.)

(On a few machines it is not necessary to press the $\boxed{=}$ key first, but it is safer to do this routinely – at least until you are certain of what your machine can do.)

Problems:

1. What does an exponent tell us?

2. Write each of the following as repeated multiplications.

   a. 5^4 = _____

   b. 17^5 = _____

   c. 8^6 = _____

   d. 125^5 = _____

   e. 3^3 = _____

3. Square each of the following numbers.

   a. 23^2 = _____

   b. 30^2 = _____

   c. 125^2 = _____

   d. 821^2 = _____

   e. 13^2 = _____

   f. 1.36^2 = _____

   g. 0.05^2 = _____

   h. 50^2 = _____

   i. 901^2 = _____

   j. 85.72^2 = _____

4. Raise each base to the indicated power.

   a. 6^5 = _____

   b. 213^3 = _____

   c. 57^3 = _____

   d. 8^4 = _____

   e. 5^5 = _____

   f. 15^3 = _____

   g. 12^3 = _____

   h. 0.01^3 = _____

   i. 5.5^4 = _____

   j. 2^8 = _____

5. Find the square root of each of the following.

   a. $\sqrt{64}$ = _____

   b. $\sqrt{225}$ = _____

   c. $\sqrt{289}$ = _____

   d. $\sqrt{576}$ = _____

   e. $\sqrt{900}$ = _____

   f. $\sqrt{69696}$ = _____

   g. $\sqrt{1234.8196}$ = _____

   h. $\sqrt{7617.7984}$ = _____

   i. $\sqrt{13675204}$ = _____

   j. $\sqrt{85.692049}$ = _____

6. Find the indicated roots if you can.

a. $\sqrt[5]{32}$ = _____

b. $\sqrt[4]{81}$ = _____

c. $\sqrt[3]{8}$ = _____

d. $\sqrt[4]{1296}$ = _____

e. $\sqrt[5]{759375}$ = _____

f. $\sqrt[9]{262144}$ = _____

g. $\sqrt[3]{0.046656}$ = _____

h. $\sqrt[4]{4096}$ = _____

i. $\sqrt[4]{65536}$ = _____

j. $\sqrt[3]{16.387064}$ = _____

7. Perform the indicated operations.

a. $(17 + 12)^2$ = _____

b. $\sqrt{146 - 137}$ = _____

c. $\sqrt{15 + 210}$ = _____

d. $(\frac{18}{90})^2$ = _____

e. $\sqrt{169 - 64}$ _____

f. $(11 + 13)^2$ = _____

g. $\sqrt{\frac{100}{25}}$ = _____

h. $\sqrt{.04 - .01}$ = _____

i. $(1.2 \cdot 2.7)^2$ = _____

j. $\sqrt{16 + 9}$ = _____

8. The Society of Automotive Engineers use the formula

$$\text{h.p.} = \frac{d^2 n}{2.5}$$

to compute engine horsepower,

where d = the diameter of the cylinder in inches

n = number of cylinders

If d = 3.8 inches and n = 8, then S. A. E. h.p. = _____ h.p.
(Hint: Square d first.)

9. In basic electricity we have the formula

$$W = I^2 R$$

where W = number of watts
I = current-flow in amps
R = resistance in ohms

If I = .015 amps and R = 10,000 ohms, then W = _____ watts.

10. In a duct system with a constant air density, the static pressure varies in proportion to the square of the fan speed.

$$\frac{P_2}{P_1} = \left(\frac{rpm_2}{rpm_1}\right)^2$$

which gives us: $$rpm_2 = rpm_1 \cdot \sqrt{\frac{P_2}{P_1}}$$

Hint: The order of operations in a problem like this:
- a. Divide P_2 by P_1.
- b. Take the square root of the result.
- c. Then multiply by rpm_1.

If $rpm_1 = 700$, $P_2 = 0.487$ inches of water, and $P_1 = 0.25$ inches of water, then rpm_2 =

_____ rpm.

(Record only the digits to the left of the decimal point.)

11. In a duct system with constant air density, the horsepower varies in proportion to the cube of the fan speed.

$$\frac{hp_2}{hp_1} = \left(\frac{rpm_2}{rpm_1}\right)^3$$

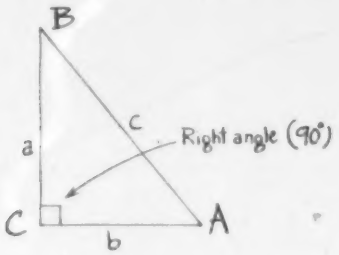
$$hp_2 = hp_1 \left(\frac{rpm_2}{rpm_1}\right)^3$$

Hint: The order of operations:
- a. Divide rpm_2 by rpm_1.
- b. Cube the result.
- c. Then multiply by hp_1.

If $hp_1 = 0.333$, $rpm_2 = 979$, and $rpm_1 = 700$, then hp_2 =

_____ h.p.

(Record only the first two digits to the right of the decimal point.)

12. 

The sides of all right triangles are related by the formula

$$a^2 + b^2 = c^2$$

(This is the Pythagorean theorem.) If we know two of the sides of any right triangle, we can find the third side by the following formulas (which we obtained by applying the rules of algebra to the above formula).

$$a = \sqrt{c^2 - b^2}$$

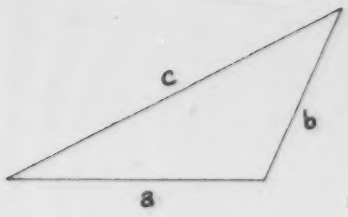
$$b = \sqrt{c^2 - a^2}$$

$$c = \sqrt{a^2 + b^2}$$

Complete the following table, using the formulas given. Do all work using the full capacity of your calculator. Use a piece of paper to record intermediate answers.

	a	b	c
(1)	9	12	
(2)		268	335
(3)	485		1261
(4)		57.0	64.6
(5)	5.32	18.24	

13.



The area of any triangle can be found using the formula

$$A = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$$

Where $s = \frac{1}{2}(a + b + c)$ – often called the semi-perimeter of the triangle – and a, b, and c are the lengths of the sides of the triangle. This formula is useful when we are unable to find the altitude (height) of the triangle, as on page 58.

In a certain triangle, a = 4.2 cm, b = 5.6 cm, and c = 7.0 cm. Find

a. $s =$ _____ cm

b. $(s - a) =$ _____ cm

c. $(s - b) =$ _____ cm

d. $(s - c) =$ _____ cm

e. $A =$ _____ cm^2

14. In basic electricity we have the formula

$$E = \sqrt{WR}$$

Where E = electromotive force in volts
W = number of watts
R = resistance in ohms.

If R = 1682 ohms, and W = 2 watts, then E = _____ volts

15. Find the floor area of a room 14.5 feet square.

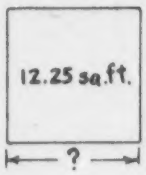
_____ sq. ft.

16. Find the area of a square lot that measures 150 feet on a side.

_____ sq. ft.

17. How long would each side of a square table have to be if the table was to have 12.25 sq. ft. of surface?

_____ ft.



18. How long would each edge of a cube have to be if each side was to have an area of 625 square inches?

_____ inches

19. What would be the length and width of a square field that covered exactly one acre?

_____ feet

(Drop all but one decimal place.)

20. If flagstone costs 55 cents a square foot, find the cost of a square flagstone patio that measures 19 feet on a side.

$_____

73

III – DECIMALS AND PERCENTS

Lesson 1 — **Rounding Off Numbers**

Objectives:

1. You should be able to round off numbers, following accepted rules.
2. You should be able to recognize when numbers should be rounded off.

Information:

Suppose you were interested in the number of miles per gallon that your car got. On a trip that covered 210 miles, your car used 8.7 gallons of gas. Dividing 210 by 8.7, you got a figure of 24.137931 mpg. Did your car actually go 24.137931 miles on each gallon?

Of course it didn't. Sometimes it did better, sometimes it did worse. Furthermore, that figure was based on 210 miles. Was it exactly 210 miles? It could have been a little more or a little less. And did you really use exactly 8.7 gallons? Maybe your tank was not absolutely full when you started to count the miles. Maybe the amount of gas replaced was not exactly 8.7 gallons.

Each figure used in calculating your average gas mileage was really only an *approximation.* Any measurement is only an approximation. Therefore an answer like 24.137931 – which *seems* to be very accurate – is *pretending* to an accuracy which it doesn't really have.

How then would you treat such a figure in real life? You would *round it off.* In this case you would round it off to the *nearest whole number,* and conclude that on your trip you got *about* 24 miles to the gallon.

Some figures need to be much more accurate than others. In scientific work, in trades like machine shop, drafting, industrial sheet metal, electronics production, and in some business situations, we might need highly accurate numbers. For other trades, and for everyday life, extreme accuracy is not usually needed. For example, a carpenter usually measures to the nearest sixteenth of an inch. A mason might measure to the nearest quarter of an inch. A nurse taking blood pressure would give the figures to the nearest 10 millimeters of mercury. A cook will even, at times, use "a pinch" of some spice.

Very often our calculators will produce numbers that are much more accurate (or that *pretend* to be more accurate) than we need. We get a string of digits, and we are not sure what to do with them all.

Suppose we divide the number 3 by the number 4. The calculator gives us 0.75, which is a reasonable figure to deal with in real life.

Suppose we divide the number 2 by the number 3. The answer is a decimal point followed by an infinite number of 6's. (0.666666666666666666....) Of course we cannot use an infinite number of sixes.

On some calculators that display 7 digits to the right of the decimal point, the answer is $0.\underbrace{6666666}_{\text{7 digits}}$,

In this case, all of the 6's after those shown in the display were simply dropped.

Some other calculators that also display 7 digits to the right of the decimal point will give the answer as 0.6666667. In this case we say that the calculator *rounded off* the last digit.

By rounding off we mean that the *last digit* (figure) in a number *that is to be kept* is:

a. increased by 1 if the digit *immediately to the right of it* is a 5, 6, 7, 8, or 9.

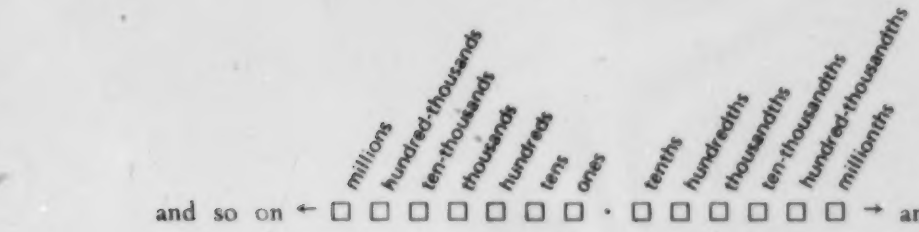
b. not changed if the digit *immediately to the right of it* is a 0, 1, 2, 3, or 4.

Example: Round off the number 24.137931 *to the nearest whole number.*

The whole-number part of this number is 24; the rest is a decimal. Since the digit *immediately to the right* of the last whole number (4) is a 1, we simply drop it (and all the figures to the right of it). The rounded-off number is 24.

Example: Round off 40.76 *to the nearest tenth*. The digit in the tenths place is 7. The digit *immediately to the right of it* is a 6. In this case we increase the 7 by 1 to make 8, and drop the 6. The rounded-off number is 49.8.

The chart shown here will remind you of the values of the different places in a number. All the places to the left of the decimal point comprise the whole-number portion. The places to the right of the decimal point comprise the decimal portion.



Here are some more examples to show you how rounding off works:

1. Round off 5732.715 to the nearest *hundredth*.

   Answer: 5732.72

2. Round off 438.05 to the nearest *hundred*.

   Answer: 400

3. Round off 543.1278 to the nearest tenth.

   Answer : 543.1

4. Round off 0.78263 to the nearest thousandth. Answer : .783

5. Round off 0.789635 to the nearest thousandth.

   Answer: 0.790

   (Do not drop that last zero. It shows that the number is correct to the nearest thousandth.)

6. Round off $532.43712 to the nearest cent.

   Answer: $532.44

A good rule to follow in using your calculator is to work with all the digits that the calculator produces, but round off the answer according to the nature of the problem.

Problems:

1. Would a carpenter be likely to drill a hole 0.573" across? Would a machinist?

2. Explain the statement that all measurements are approximations.

3. If you were measuring a room for a wall-to-wall carpet and got 4.33 metres by 3.67 metres, how many square metres of carpet would you need?

   _____ m^2

   How many square metres would you *order?* _____ m^2

4. Round these off to the nearest cent.

   a. $12.7195 _____
   b. $92.3055 _____
   c. $6.2976 _____
   d. $10.0049 _____
   e. $15.18099 _____
   f. $100.5945 _____
   g. $100.5955 _____
   h. $24.005 _____
   i. $13.1509 _____
   j. $.9951 _____

5. On your Federal income tax return, you are permitted to round off each amount you enter, to the nearest whole dollar. How would you enter these amounts?

   a. $59.63 _____
   b. $117.49 _____
   c. $2000.55 _____
   d. $17.75 _____
   e. $152.01 _____
   f. $420.98 _____
   g. $33.33 _____
   h. $14.49 _____
   i. $14.50 _____
   j. $199.65 _____

6. Round off each of the following to the place indicated.

   a. 53.245 – hundredths _____
   b. 0.79213 – thousandths _____
   c. 17.21367 – ten thousandths _____
   d. 259.53714 – tenths _____
   e. 67.7983 – hundredths _____
   f. 7,831,286 – millions _____
   g. 75.998 – hundredths _____
   h. 99.527 – ones _____
   i. 0.6132719 – ones _____
   j. 4532.178 – hundredths _____

7. Find the volume of a box 2.7 cm × 3.4 cm × 1.8 cm, on your calculator.

   _____ cm^3

   What would be a reasonable answer to give? _____ cm^3

Lesson 2 — **Fractions to Decimals**

Objectives:

1. You should be able to use your calculator to express a given fraction as a decimal.
2. You should know from memory the decimal values of the commonly used fractions presented in this lesson.
3. You should be able to use your calculator to add, subtract, multiply, and divide with fractions and mixed numbers.

Information:

In the United States we have a monetary system based upon the decimal system. On December 23, 1975, President Gerald R. Ford signed the Metric Conversion Act (Public Law 94-168). According to this law the United States is now engaged in a voluntary changeover to the metric system of weights and measures. Therefore, our new system of weights and measures is also based upon the decimal system. These are just two examples that show the importance of the use of the decimal system which, fortunately, is also the language of the hand calculator.

A common fraction has two parts. These are the numerator and the denominator.

$$\frac{\text{numerator (top)}}{\text{denominator (bottom)}}$$

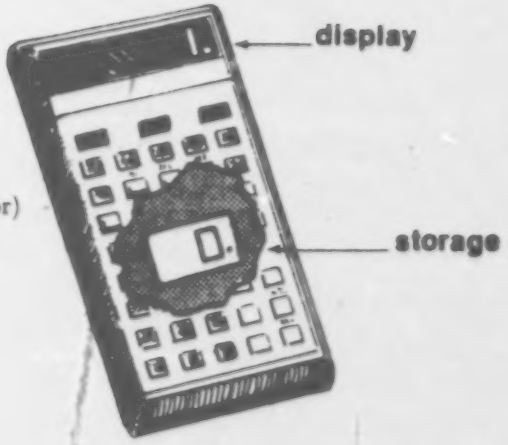
Each common fraction can be thought of as a division problem where the numerator is the dividend and the denominator is the divisor.

Therefore, the common fraction $\frac{3}{4}$ can be thought of as 3 divided by 4, or $3 \div 4$.

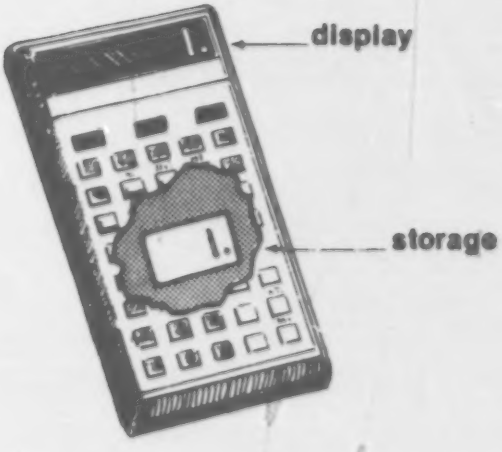
Example: Express ½ as a decimal.

$\frac{1}{2}$ means $1 \div 2$

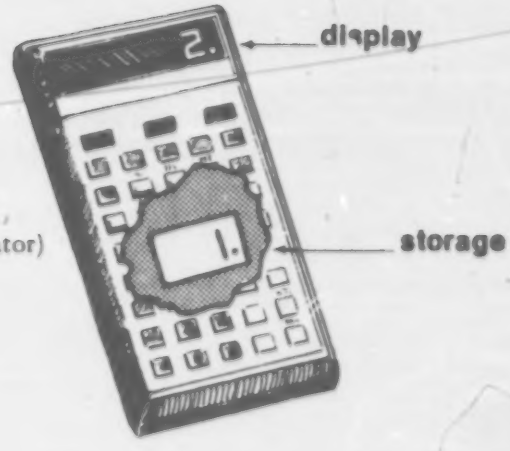
Step 1 – Enter 1 (numerator)



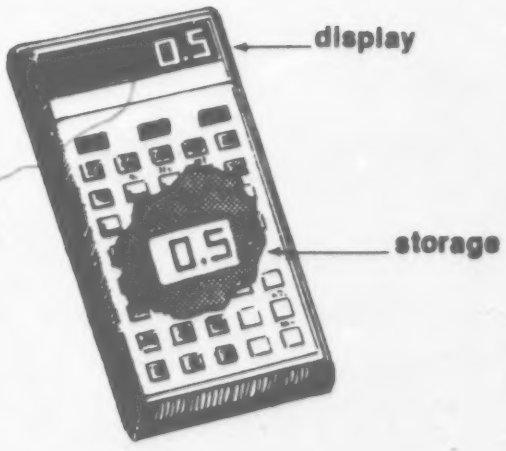
Step 2 – Press [÷]



Step 3 – Enter 2 (denominator)



Step 4 – Press [=]



Therefore, $\frac{1}{2} = 0.5$

The following table presents a list of commonly used fractions. The student is urged to memorize the decimal equivalent of each.

$\frac{1}{8}$	0.125
$\frac{1}{4}$	0.25
$\frac{3}{8}$	0.375
$\frac{1}{2}$	0.5
$\frac{5}{8}$	0.625
$\frac{3}{4}$	0.75
$\frac{7}{8}$	0.875

$\frac{1}{5}$	0.2
$\frac{2}{5}$	0.4
$\frac{3}{5}$	0.6
$\frac{4}{5}$	0.8

$\frac{1}{3}$	.333
$\frac{2}{3}$	.667

These decimal values were rounded off to the thousandths place.

Example – Addition

$$5\frac{1}{2} + \frac{2}{5} + 1\frac{7}{10} = ?$$

Substitute the decimal equivalent for each fraction.

$$5.5 + 0.4 + 1.7 = ?$$

Use your calculator to find the sum.

Example – Multiplication

$$5\tfrac{1}{3} \cdot 6\tfrac{1}{8} \cdot \tfrac{3}{4} = \ ?$$

Substitute the decimal equivalent for each fraction.

$$5.333 \cdot 6.125 \cdot 0.75 = \ ?$$

Use your calculator to find the product.

The two examples above use fractions that you have memorized or will soon memorize. If you must work with a fraction you have not memorized, then you must first find its decimal equivalent before you can proceed with the indicated operation.

Example: $\frac{1}{16} \times 472 = \ ?$

ENTRY	DISPLAY	STORAGE
1	1.	0.
÷	1.	1.
16	16.	1.
×	0.0625	0.0625
472	472.	0.0625
=	29.5	29.5

Example: $\frac{1}{6} + \frac{7}{12} = \ ?$

Using the calculator, we find that $1 \div 6 = 0.1666666$
We round this off to 0.167.
Dividing 7 by 12, we get 0.5833333, which we round off to 0.583.
Adding the two results give us

$$\tfrac{1}{6} + \tfrac{7}{12} = 0.75$$

Problems:

1. Complete the following tables from memory.

	Decimal
$\frac{1}{8}$	
$\frac{1}{4}$	
$\frac{3}{8}$	
$\frac{1}{2}$	
$\frac{5}{8}$	
$\frac{3}{4}$	
$\frac{7}{8}$	

	Decimal
$\frac{1}{5}$	
$\frac{2}{5}$	
$\frac{3}{5}$	
$\frac{4}{5}$	

	Decimal
$\frac{1}{3}$	
$\frac{2}{3}$	

Round off to the thousandths place.

2. Give the decimal equivalent for each of the following from memory.

a. $\frac{2}{5}$ = _____
b. $\frac{1}{3}$ = _____
c. $\frac{7}{8}$ = _____
d. $\frac{1}{5}$ = _____
e. $\frac{3}{4}$ = _____
f. $\frac{1}{8}$ = _____
g. $\frac{2}{3}$ = _____
h. $\frac{7}{10}$ = _____
i. $\frac{3}{5}$ = _____
j. $\frac{1}{2}$ = _____
k. $\frac{3}{10}$ = _____
l. $\frac{4}{5}$ = _____
m. $\frac{1}{4}$ = _____
n. $\frac{3}{8}$ = _____
o. $\frac{5}{8}$ = _____
p. $\frac{9}{10}$ = _____

3. U. S. Government bonds are traded in 32nds of a dollar. Find the exact value of:

a. 3/32 _____
b. 7/32 _____
c. 19/32 _____
d. 31/32 _____
e. 16/32 _____

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4. Mr. Gottrocks bought three Government bonds. He paid at the rate of 98 5/32, 99 13/32, and 98 27/32 for them. What was the average rate he paid? (to the nearest cent)

_____

5. Express each of the following fractions as a decimal. Do any that you can mentally. Do the rest on your calculator. Round your answers off to 3 decimal places if necessary.

a.	$\frac{50}{100}$ =	_____	f.	$\frac{4}{32}$ =	_____	k.	$\frac{2}{4}$ =	_____
b.	$\frac{25}{75}$ =	_____	g.	$\frac{11}{12}$ =	_____	l.	$\frac{30}{50}$ =	_____
c.	$\frac{3}{16}$ =	_____	h.	$\frac{15}{25}$ =	_____	m.	$\frac{243}{1000}$ =	_____
d.	$\frac{10}{40}$ =	_____	i.	$\frac{35}{100}$ =	_____	n.	$\frac{79}{100}$ =	_____
e.	$\frac{10}{16}$ =	_____	j.	$\frac{4}{6}$ =	_____	o.	$\frac{7}{12}$ =	_____

6. Perform the indicated operations. Work with 3 decimal places where necessary and round off all final answers to the hundredths place.

a.	$\frac{1}{2} + \frac{1}{4} + \frac{1}{3} =$	_____	f.	$6\frac{3}{4} - 4\frac{1}{5} =$	_____
b.	$5 \cdot 6\frac{7}{8} =$	_____	g.	$\frac{1}{2} \cdot 342\frac{1}{2} =$	_____
c.	$\frac{3}{5} \div \frac{1}{4} =$	_____	h.	$4\frac{1}{3} \cdot 18 =$	_____
d.	$\frac{7}{8} \cdot \frac{2}{3} \cdot \frac{1}{2} =$	_____	i.	$13 - 7\frac{4}{5} =$	_____
e.	$11\frac{5}{8} \div 3 =$	_____	j.	$\frac{7}{10} \cdot 14 =$	_____

7. Phineas Chittleworth was a bachelor. When he died, his estate amounted to $46,789. His will stated that half his estate was to go to his favorite nephew Phineas II, one-fifth to his niece Phinea, and one-tenth to his friend Rhoda. The balance was to go to the Country Pet Home, to be used for the care of his six cats as long as they lived. How much money did the pet home receive?

_____

(Hint: It should not be necessary to work out the dollar amounts for his relatives – work with the fractions only until the final step.)

8. Buzz Sohr the carpenter had to cut two boards, each 9¾" long, out of a 12-foot board. If each cut wasted 1/16", how much of a board would be left over?

_____ inches

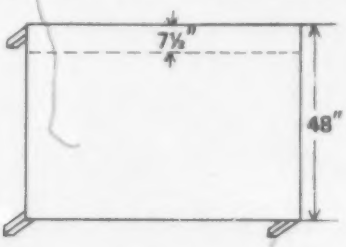
or _____ feet

9. The Ms. Perfect dress factory had the following production scheduled for model #921:

110	in	size	8 @	2 7/8	yards	per dress
175	"	"	10 @	3	"	" "
250	"	"	12 @	3 1/8	"	" "
190	"	"	14 @	3 1/8	"	" "
110	"	"	16 @	3 1/4	"	" "
						Total yardage

Figure out the yardage needed for each size and the total yardage to be ordered.

10. How many pieces of finished lumber 7½" wide are needed to make the top of this table?



11. Steve Vargas decided to put a new roof on his garage. The roof was a standard gable type, with two identical sides, each side measuring 9' X 20'. If a bundle of shingles covers 33 1/3 square feet, how many bundles would he have to buy?

12. The bundles of shingles were on sale for $5.77 each, reduced from the regular price of $7.69. How much would Steve save by buying them on sale?

III – DECIMALS, PERCENTS

Lesson 3 — **Percentage**

1. You should be able to express a given number in percent form.

2. You should be able to express a given common fraction as a decimal and as a percent.

3. You should be able to express a given percent as a decimal.

4. You should be able to solve various types of problems dealing with percentages, using the [%] key on your calculator.

Information:

Percents are used in a great many ways in business and in ordinary, everyday living. What is percent?

"Percent" actually means "for each 100." A percent is *a common fraction with 100 as its denominator.*

Examples:

$$2\% = 2 \text{ out of } 100 = \frac{2}{100}$$

$$25\% = 25 \text{ out of } 100 = \frac{25}{100}$$

$$100\% = 100 \text{ out of } 100 = \frac{100}{100}$$

$$131\% = 131 \text{ compared to } 100 = \frac{131}{100}$$

Suppose we are told that 741 students in the student body of 1346 are girls. These figures do not *mean* too much to the average person. But if we know that *55%* are girls, we have a much better picture of the makeup of the student body. Where did we get the 55%?

$$741 \text{ out of } 1346 \text{ is } \frac{741}{1346}$$

Dividing the fraction on the calculator, we get

0.55052, which rounds off to 0.55.

To change the 0.55 to percent, we *multiply by 100* (and add the percent sign).

$$0.55 \times 100 = 0.55\% = 55\%$$

We have not changed the value of 0.55 in any way. We have *multiplied it by 100* and then, by using the percent sign, have in effect *divided* by 100. Multiplying and dividing a number by the same quantity will not change the number.

Examples:

1. Express .25 as a percent.
   $0.25 \times 100 = 0.25\% = 25\%$

2. Express .03 as a percent.
   $0.03 \times 100 = 3\%$

3. Express .085 as a percent.
   $0.085 \times 100 = 8.5\%$ (or $8\frac{1}{2}\%$)

4. Express 3 as a percent.
   $3 \times 100 = 300\%$

If we are given a number expressed as a percent, then we can write this number in decimal form by reversing the above process. That is, we *divide* by 100 (move the decimal point two places to the *left*).

Examples:

$200\% = 200 \div 100 = 200. = 2.$
$53.5\% = 0.535$
$1.3\% = 0.013$
$50\% = 0.50$

As we saw in the student-body example above, if we have a common fraction to express as a percent, we first change that fraction to decimal form and then multiply by 100.

Example: Express $\frac{7}{8}$ as a percent

$$\frac{7}{8} = 0.875 = 87.5\%$$

Some students are not sure whether to multiply or divide by 100 in changing from decimal to percent and vice versa. These students might find it helpful to think about our coins. For example,

$$\frac{1}{4} \text{ (of a dollar)} = \$.25 = 25\% \text{ of a dollar.}$$

It is clear that the decimal point must be moved to the *right* – to *multiply* – in changing from the decimal to the percent.

To find a given percent of a number, we multiply the percent *expressed as a decimal* by the number.

Example: Find 25% of * 64

Step 1 – Rewrite 25% as 0.25

Step 2 – Multiply 0.25 · 64

ENTRY	DISPLAY	STORAGE
.25	0.25	0.
[X]	0.25	0.25
64	64.	0.25
[=]	16.	16.

To find *what percent one number is of another,* make a fraction with the first as numerator and the second as denominator. Divide the fraction on the calculator and then move the decimal point two places to the right to get the percent.

Example: A test had 40 questions, and Cindy got 27 correct. What percent did she score?

ENTRY	DISPLAY	STORAGE
27	27.	0.
[÷]	27.	27.
40	40.	27.
[=]	0.675	0.675

Cindy scored 67.5%, which would be rounded off to 68%.

* The word "of" implies multiplication.

Many calculators have special percent keys, [%] . With such a calculator the first problem above can be solved without the need to change the percent to a decimal.

Example: Find 25% of 64.

ENTRY	DISPLAY	STORAGE
64	64.	0.
[X]	64.	64.
25	25.	64.
[%]	16.	16.

Two types of problems that deal with percentage are markup and discount problems. **Markup** is an amount *added to the cost price* to determine the selling price.

Suppose as a store owner you are able to buy an item at the cost price of $25 and wish to sell it at a 30% markup. What is the selling price?

Solution (assuming no [%] key on calculator):

Step 1 – Determine 30% of 25 – the markup.

ENTRY	DISPLAY	STORAGE
25	25.	0.
[X]	25.	25.
.30	0.30	25.
[=]	7.5	7.5

Step 2 – Add the markup to the cost price.

ENTRY	DISPLAY	STORAGE
result above	7.5	7.5
[+]	7.5	7.5
25	25.	7.5
[=]	32.5	32.5

Therefore, the selling price will be $32.50.

Many of the calculators with the percent key, [%] , have a procedure for *automatic markup*. We will illustrate this by doing the above problem again, using the [%] key on such a machine.

ENTRY	DISPLAY	STORAGE
25	25.	0.
[+]	25.	25.
30	30.	25.
[%]	7.5	25.
[=]	32.5	25.

A **discount** is an amount that will be *deducted* from a price.

Suppose an item has a regular price of $4.99 and today is being marked down (discounted) by 20%. What is the selling price?

Solution (assuming no [%] key):

Step 1 – Determine 20% of $4.99.

ENTRY	DISPLAY	STORAGE
.20	0.20	0.
[×]	0.2	0.2
4.99	4.99	0.2
[=]	0.998	0.998

Round off to $1.00

Step 2 – Subtract the discount from the list price.

ENTRY	DISPLAY	STORAGE
4.99	4.99	0.
[-]	4.99	4.99
1	1.	4.99
[=]	3.99	3.99

Therefore, the selling price is $3.99.

Again the calculator with the [%] key – particularly if it has the automatic markup and discount – can save us time and effort. Here is the above problem using such a machine.

ENTRY	DISPLAY	STORAGE
4.99	4.99	0.
[-]	4.99	4.99
20	20.	4.99
[%]	0.998	4.99
[=]	3.992	4.99

Round this off to $3.99.

Sometimes you want to know the percent increase or decrease of one number compared with another. When working these problems,

1. Find the difference in amount
2. Divide this by the original number to get a decimal
3. Convert the decimal to a percent.

Example: Kevin's pay was raised from $2.65 to $3.00 an hour. What was the *percent of increase?*

	ENTRY	DISPLAY	STORAGE
	3	3.	0.
(1)	[-]	3.	3.
	2.65	2.65	3.
	[÷]	0.35	0.35
(2)	2.65	2.65	0.35
	[=]	0.1320755	0.1320755

(3) Change the answer to a percent, and you get (rounded off) 13.2%.

Problems:

1. A percentage is a common fraction with _____ as its denominator.

2. In order to express a given number as a percentage we _____ the decimal form of that number by 100. (Move the decimal point ____ places to the _____ .)

3. In order to express a given percent as a decimal we _____ the percentage by 100. (Move the decimal point ___ places to the _____.)

4. Express each of the following as a percent.

a.	0.25 =	_____	%	f.	$\frac{3}{8}$ =	_____	%	k.	0.0525 =	_____	%
b.	0.05 =	_____	%	g.	$\frac{4}{5}$ =	_____	%	l.	$\frac{79}{100}$ =	_____	%
c.	0.0312 =	_____	%	h.	0.104 =	_____	%	m.	0.002 =	_____	%
d.	2.53	_____	%	i.	4 =	_____	%	n.	00004 =	_____	%
e.	0.065 =	_____	%	j.	6.15 =	_____	%	o.	$\frac{13}{100}$ =	_____	%

5. Express each of the following percents as decimals.

a.	62.5% =	_____		f	3%	_____		k.	$3\frac{1}{3}$% =	_____	
b.	50% =	_____		g.	1.5% =	_____		l.	.1% =	_____	
c.	25% =	_____		h.	2½% =	_____		m.	1200% =	_____	
d.	75% =	_____		i.	500% =	_____		n.	37.5% =	_____	
e.	87.5% =	_____		j.	100% =	_____		o.	10% =	_____	

6. Find the indicated percentage of each of the following. (Round off to two decimal places where necessary.)

Example: 3% of 50 = 1.5

a. 75% of 30 = _____

b. 50% of 596 = _____

c. 2% of 60 = _____

d. 1.5% of 75 _____

e. 100% of 130 = _____

f. 200% of 71 = _____

g. 25% of 5.99 = _____

h. 0.4% of 600 = _____

i. 80% of 100 = _____

j. 28.4% of 89.32 = _____

7. Complete the following table of markups. Give answers in dollars and whole cents.

	Cost	Percent of markup	Selling Price
a.	$ 26.75	7%	
b.	1532.99	30%	
c.	59.90	4%	
d.	150.00	120%	
e.	10.01	36%	
f.	.62	7.5%	

8. Complete the following table of discounts. Give answers in dollars and whole cents.

	Cost	Percent of Discount	Selling Price
a.	$ 399.99	25%	
b.	14.50	33%	
c.	9.95	40%	
d.	3.75	5%	
e.	2500.00	18%	
f.	15.95	13.7%	

9. There are two types of simple interest, exact and ordinary. Exact interest is calculated on the basis of 365 days in a year, while ordinary interest is calculated on the basis of only 360 days in a year. The formulas are as follows:

$$E = \frac{Prd}{365} \qquad \text{and} \qquad O = \frac{Prd}{360}$$

where

E = exact interest
O = ordinary interest
P = principal
r = interest rate per year
d = number of days

a. If P = $14,600, r = 6%, and d = 120 days, then E = $_____ and O = $_____

b. How much must be repaid in each case at the end of 120 days?

E = $_____
O = $_____

c. How much must be repaid on a loan of $2400 for 75 days at 8¼% (ordinary) interest? (Hint: Work out the interest first.)

_____

10 In one year the U. S. automobile industry sold 8,971,242 cars. The next year the industry sold 8,701,865 cars. By what percent did sales *increase or decrease* the second year?

_____ _____ by _____ %

11. The Splurge family at a restaurant received the following check for four dinners.

Bluefish	$4.95
St. cabbage	3.95
Port. steak	6.95
Seafood plat.	5.95
3 coffee	.75
1 tea	.25
2 cheese cake	1.50
	25.30
5% tax	1.27
	$26.57

Check the check and correct it if necessary. How much of a tip (approximately) should the family leave if they wish to leave 15% of the food bill (not including tax)?

_____ _____

12. The workers in a plant were given a 6.5% raise. Fill in the new pay rates for the following workers.

	Old rate	New rate
B. McDonald	2.67	_____
L. Carey	3.39	_____
D. Masiello	5.95	_____
V. Rodriguez	3.86	_____
T. Smith	4.05	_____

13. According to the Federal Energy Department, most automobiles get about 21% more miles per gallon on the highway at 55 miles per hour than at 70 mph. Assuming this applies to your car, which averages 23 miles per gallon at 70 mph, about how many miles per gallon would you get if you drove at 55 mph?

_____

14. The following students took a test consisting of 25 problems. Work out the percent of right answers that each student scored.

Joe	20 correct	_____
Sally	18 correct	_____
Bill	21 correct	_____
John	13 correct	_____
Joan	25 correct	_____

15. In a performance test for gas mileage, a Toyota Corolla scored 29 miles per gallon at a steady 60 miles per hour and 38 mpg at a steady 40 mph. Compare the gas mileage at 60 mph *with the mileage at 40 mph.* (Use a percent figure.)

_____

16 Constant-Speed Mileage of Four Cars

	Toyota Corolla		Ford Pinto		AMC Gremlin		Pontiac Astre	
40 mph	38	–	28	–	27	–	34	–
50 "	35		25		24		31	
60 "	29		23		20		27	

Above is a table comparing the gasoline mileage figures of four different models of cars. In the empty spaces, fill in the percent by which the mileage at 50 and at 60 *decreased from the mileage at 40 mph.* Round off to the nearest whole percent.

(Hint: Subtract each figure from the mileage at 40 before computing the percent.)

Then answer the following questions:

a. Which car's mileage rate held up best at 50 mph? _____

b. Which car's mileage rate held up best at 60 mph? _____

c. Which car is actually most economical to run, from the point of view of gasoline cost? _____

d. Explain the difference between (b) and (c).

17. The ad said: "One-week special on permanent waves."

Balsam Plus	*Regularly*	$12.50	*This week only*	$10.00
Helene Glamour	*Regularly*	$15.00	*This week only*	$11.50
Creme de Menage	*Regularly*	$25.00	*This week only*	$15.00

Which wave had the largest markdown, in terms of percent? _____

Is that wave necessarily the best buy? Explain.

18. The efficiency of a machine is given by the formula:

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}}$$

Efficiency is generally stated in percent.

The input of an electric motor is 2610 watts, which is equal to 3.5 horsepower. If the motor delivers 3 horsepower to a load, what is the efficiency (in percent) of this motor? _____

19. Last year the Eatwell Diner made a net profit of $18,000 for its owner. This year its net profit came to $22,500. By what percent did its profit *increase* this year? _____

20. Down the block from the Eatwell Diner, the Colonial Tavern made a net profit this year of $32,000 for its owner, compared to $30,000 last year. What was the percent of increase this year? _____

21. Would you rather own the Eatwell Diner or the Colonial Tavern? Explain.

_____

22. Three operators turned out the same machined part one week.
    Operator A had 12 rejects out of 460 parts.
    Operator B had 19 rejects out of 550 parts.
    Operator C had 4 rejects out of 325 parts.
    Which operator did the most accurate work? _____

    Which operator turned out the greatest number of acceptable parts? _____

23. The Fashion Figure Dress Shop bought a line of garments at $17.75 each and sold them at $33.95. What was the percent of markup?

_____

24. J. C. Nickel department store advertised "up to 40% off on all draperies." Sally saw a pair she liked, regularly priced at $22.95, on sale for $18.95. What was the actual percent of reduction?

_____

25. A layout artist for a printshop had a photograph that was 8½ inches wide to be included in an ad. She needed it reduced in width to 5 inches. What percent reduction should she indicate?

_____

26. A $5000 bond issued by the Electrigas Company will pay 7.85% interest each year for 24 years. If the owner keeps the bond for its entire "life," how much interest will the owner collect?

_____

27. a. Steve Slye enjoyed buying up "distress" goods and selling them at a flea market. He bought five dozen house plants for $21.00 and managed to sell most of them, getting $49.50 in all. What percent of "markup" did he make on the cost?

_____

b. If it cost Steve $16 to rent the space at the flea market, what percent did he actually make on his costs?

_____

28. The Blue Sox ball team won 73 and lost 89 games. What was its percentage of won games to the total number played?

_____

29. Money in a savings bank earns *compound* interest. That is, interest is paid on the money deposited (the principal) and also *on the interest* credited to the account. If a bank were to pay 5¼% a year compounded annually (each year), figure out how much $100 would come to in 5 years. (Hint: this is the same as automatic markup. Repeat your operation five times.)

_____

30. Suppose the above bank compounded the interest every 3 months (quarterly).

a. What percent of interest would be paid each compounding period? _____

b. Compute the amount that the $100 would come to in 1 year. (Hint: 1 year = 4 compounding periods)

_____

31. The Precision Ball Bearing Company received a supply of steel from the Ramrod Steel Company. The bill came to $4,290. The company decided to take advantage of the 2% discount for paying within 10 days. How much did the company pay?

_____

32. In some lines of business, retail dealers routinely receive what are called trade discounts. This means a certain discount is taken on the stated price, then a second discount is taken on the reduced price; sometimes a third discount is taken on the twice-reduced price. The Hallmark Department Store bought a supply of silverware from the Gotham Silverware Co. The bill came to $2,049.50, less trade discounts of 40%, 25%, and 5%. How much did the Hallmark actually pay?

_____

Based on the billed amount, what percent of simple discount would have given the same result as the 40-25-5 trade discount?

_____

33. Phil Stryker, the baseball hero, scored 206 hits out of 623 official times at bat. What was his batting average?

_____

If this were measured in percent instead of decimals, what percent of successful hits did he have?

_____

34. Jim Olds traded in his car after two years and was allowed $4055 for it. If it had cost him $5400, what was the percent of depreciation in the two years?

_____

35 Marilyn bought her favorite brand of peanut butter when she went shopping, and paid 89 cents for an 18 oz. jar. When she got home she noticed that the previous jar had a sticker on it for 79 cents. What percent had the peanut butter gone up in price?

_____

36. Mr. D. Linquent's business was doing poorly, and he did not pay his property tax of $2750 when it was due. The city charged 9% annual interest on taxes paid after the due date. How much did Mr. Linquent have to pay if he paid the tax (with interest) 7 months late?

_____

IV – SPECIAL FEATURES

Lesson 1 — **Multiplication and Division by a Constant**

Objective:

1. You should be able to multiply and divide by a constant number.

2. You should be able to multiply by a constant percent.

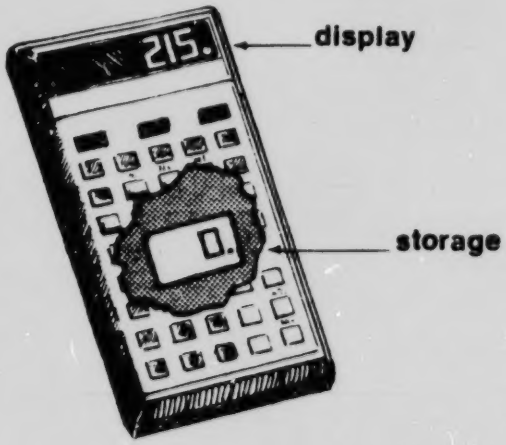
Information:

Most calculators have a feature whereby a number can be fixed in storage and used repeatedly to multiply or divide by any number entered into the display. On some calculators a constant key or switch must be pressed, while on others the process is carried out by the order in which keys are pressed.

Suppose we wanted to multiply each of the numbers 215, 1532, 312, and 25 by 37. This is multiplication by a constant. The constant is 37.

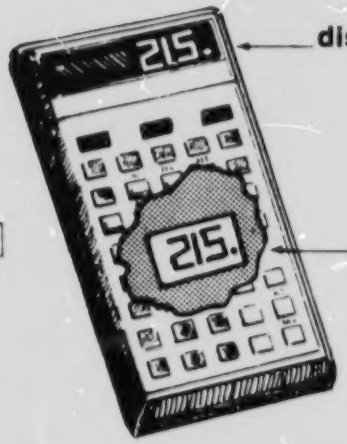
Switch to constant position (if necessary). The following is the procedure followed on many machines.

Step 1 – Enter 215*



* On some calculators it is necessary to put the constant number (here 37) into the machine *first* for constant multiplication. Check your instruction booklet or experiment yourself with simple numbers (2 X 5, 3 X 5, etc.) to find out which way your own calculator works.

Step 2 – Press [X]

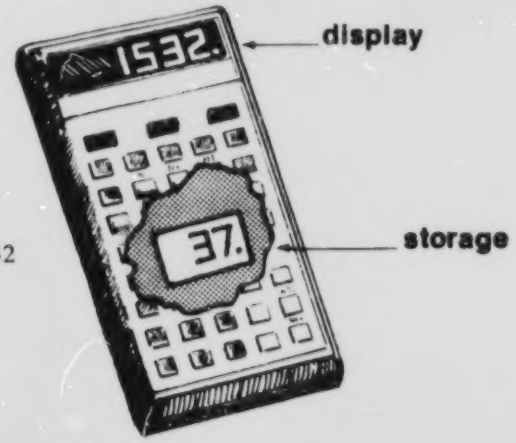


Step 3 – Enter 37

Step 4 – Press [=]

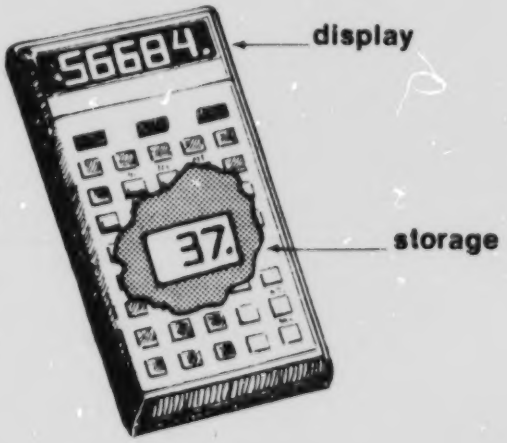
Record first answer, 7955. Notice that the answer was not placed in the storage register. Instead, the second entry was moved from the display to the storage. This is the major difference between normal operation and constant operation. Those calculators that do not have a constant key but can still be used for constant multiplication and division always keep one or the other multiplier numbers in the storage register. See the footnote on page 99.

Step 5 – Enter 1532



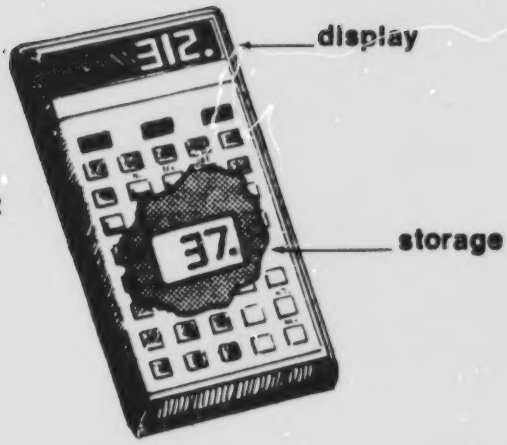
By the principle of destructive read-in, the previous answer, 7955, is erased. The calculator in the constant operation remembers that the only function key pressed so far is multiplication. It will continue to multiply the number in the display by the number in the storage.

Step 6 – Press [=]

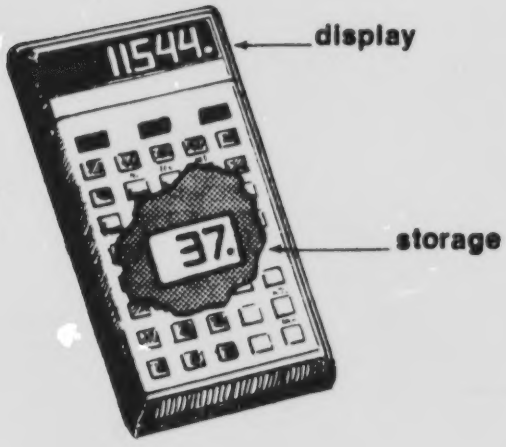


Record the answer 56,684.

Step 7 – Enter 312

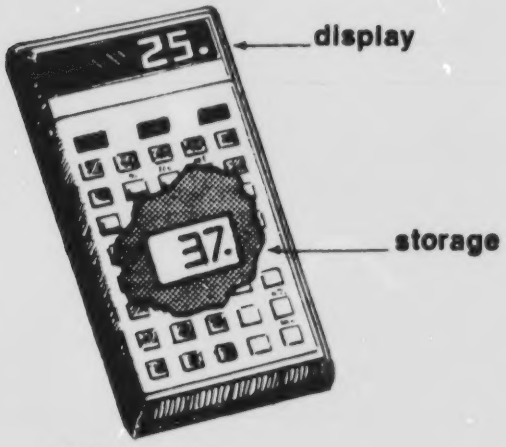


Step 8 – Press [=]



Record the answer 11,544.

Step 9 – Enter 25

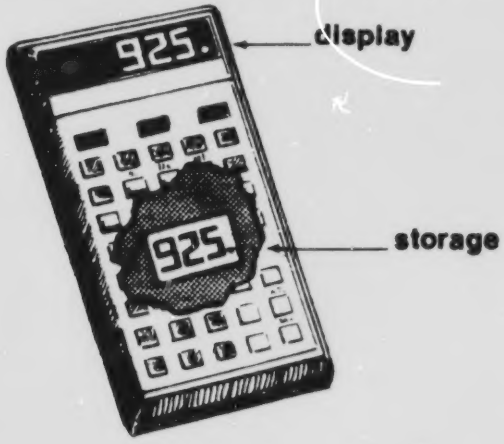


Step 10 – Press [=]

Record the answer 925.

Notice that, throughout the entire procedure, the only *operation* that was entered was the [X] in step 2. Suppose that we were to press the addition key at this point.

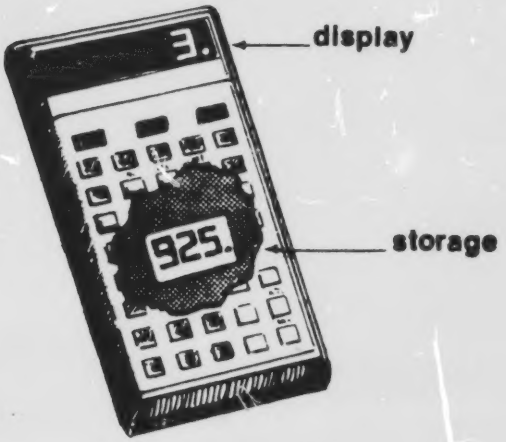
Press [+]



The pressing of a function key will cause the number in the display to be duplicated in the storage, erasing the constant. The machine will simply continue operating on the last answer, 925. For example, if we now enter a 3 – –

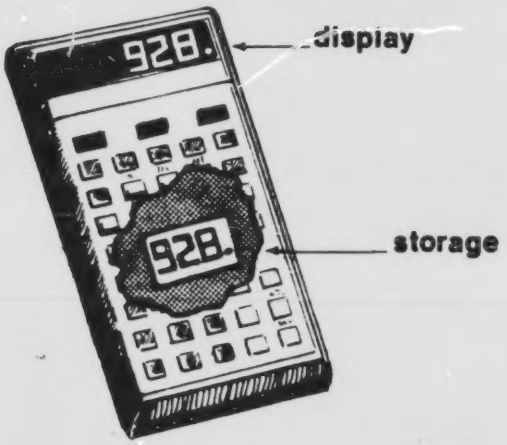
103

Enter 3



the machine will add the 3 to the 925.

Press [=]



We can also erase the constant in storage by pressing the [C] key.

On those calculators where the constant multiplier must be the *first* number pressed (footnote, page 99), the rest of the procedure is exactly the same. (In those calculators the *first* number entered is put into the storage and stays there.)

Division by a constant works the same as multiplication, except that the function key pressed in step 2 is the [÷] instead of the [X] .

Example: Division by a constant.

Divide 7955, 56684, 11544, and 925 by 37.

Switch to the constant position if necessary.

ENTRY	DISPLAY	STORAGE
7955	7955.	0.
[÷]	7955.	7955.
37	37.	7955.
[=]	215.	37.
56684	56684.	37.
[=]	1532.	37.
11544	11544.	37.
[=]	312.	37.
925	925.	37.
[=]	25.	37.

Pressed only once

Constant divisor

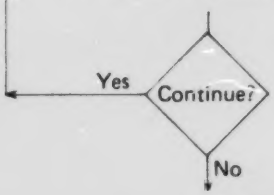
Therefore,

$$7955 \div 37 = 215$$
$$56684 \div 37 = 1532$$
$$11544 \div 37 = 312$$
$$925 \div 37 = 25$$

In summary, we will state the following sequence of steps in order to multiply or divide by a constant.

Multiplication

1. Switch constant key (if necessary).
2. Enter first number (or constant multiplier).
3. Press [X]
4. Enter number to be used as the constant multiplier (or first of the varying numbers).
5. Press [=]
6. Record answer in display register.
7. Enter next number to be multiplied by the constant.
8. Press [=]
9. Record answer.

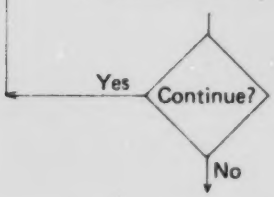


10. Press clear, [C] , or another function key to discontinue.

Division

Notice that this is exactly the same as multiplication, except in step 3 we will press the division key, [÷]

1. Switch constant key (if necessary).
2. Enter first number.
3. Press [÷]
4. Enter number to be used as the constant divider.
5. Press [=]
6. Record the answer in the display register.
7. Enter the next number to be divided by the constant.
8. Press [=]
9. Record answer.



10. Press clear, [C] , or another function key to discontinue.

Suppose that an employer has given each of his employees a 7% raise. How much more will each of the following employees receive each year?

Employee #	Yearly Salary	7% Raise
1	$12,530	
2	9,750	
3	15,800	
4	13,085	
5	11,800	

To find the amount of each employee's raise we will multiply each of the given salaries by the same number, 7% or .07. Many calculators allow the user to multiply by a constant percentage. On this type of calculator the answers to the above problem will be found as follows:

ENTRY	DISPLAY	STORAGE	
12530	12530.	0.	
[X]	12530.	12530.	
7	7.	12530.	
[%]	877.1	7.	
9750	9750.	7.	
[%]	682.5	7.	
15800	15800.	7.	
[%]	1106.	7.	Constant
13085	13085.	7.	
[%]	915.95	7.	
11800	11800.	7.	
[%]	826.	7.	

Therefore,
7% of $12,530 = $ 877.10
7% of $ 9,750 = $ 682.50
7% of $15,800 = $1106.00
7% of $13,085 = $ 915.95
7% of $11,800 = $ 826.00

Notice that the procedure for multiplying by a constant percentage is the same as multiplying by any constant, except that we press the [%] key instead of the [=] key.

On those calculators that do not permit multiplying by a constant percent by means of the [%] key, the user need only *express the percent as a decimal* and proceed as in ordinary multiplication with a constant multiplier. For example, the above 7% could have been entered into the machine as 0.07.

Problems:

1. Does your calculator have the constant multiplication and division feature?

2. If the answer to question 1 is yes, then must you switch on a constant key before starting multiplication or division by a constant? Which number do you put in first – the constant or the variable – for multiplication? for division?

3. Multiply each of the numbers below by 5; then by 2.54.

	(a) × 5	(b) × 2.54
20	_____	_____
5.63	_____	_____
0.75	_____	_____
105	_____	_____
3.14	_____	_____

4. Divide each of the numbers below by 2.7; then by 55. Round off your answers to 3 decimal places.

	(a) ÷ 2.7	(b) ÷ 55.
1.8	_____	_____
5	_____	_____
14.3	_____	_____
0.073	_____	_____
207.2	_____	_____

5. In a certain state the sales tax is 5%. Find the sales tax on each of the following items. Round off to the nearest penny.

Item Number	Selling Price	5% Sales Tax	
1	$ 6.25		
2	10.80		
3	59.95		
4	.65		
5	4.99		
6	9.99		
7	27.50		
8	42.50		
9	25.00		
10	106.30		
11	2850.00		
12	995.00		
13	3.99		
14	1.95		
15	.39		

6. It is not possible on most calculators to use the constant operation for automatic markups. A few moments' thought, however, should help you see that multiplying a number by *1 plus the markup* (in decimal form) will give you the same result as *adding the markup amount to the original figure.* For example – the selling price of an item that cost $2.00, to which a 25% markup was added, is $2.00 X 1.25 = $2.50.

   a. Using the above method, work out the final price to be charged – including the tax – for each item in problem 5. Use the last column for your answers. Round off each answer to the nearest cent.

   b. Check your answers by adding the tax to each selling price. Do this mentally or on your calculator.

7. Figure the price of each item below if the *price per dozen* is as shown. If the price has to be rounded off, *add* a cent for *any* fraction of a cent (as would be done in a store).

a.	$28.66	_____	f.	$ 5.00	_____
b.	13.50	_____	g.	35.50	_____
c.	6.75	_____	h.	12.97	_____
d	12.00	_____	i.	42.40	_____
e.	18.16	_____	j.	18.00	_____

8. You are a clerk in a store and must mark down some of the merchandise on sale at 20% off. Write the new price for each item. Round off to the nearest cent. (Hint: Figure out what decimal to multiply the original price by, and use constant multiplication.)

a.	$39.95	_____	f.	$74.90	_____
b.	15.50	_____	g.	32.50	_____
c.	24.75	_____	h.	10.49	_____
d	18.00	_____	i.	12.95	_____
e.	44.85	_____	j.	20.00	_____

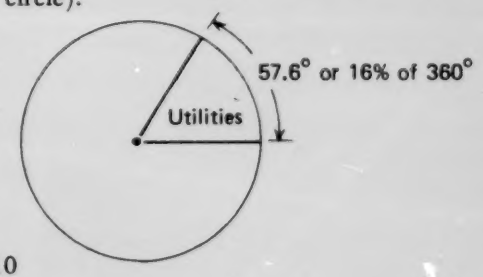
9. Patterns for metal castings must be made larger than the finished product is to be. A shrinkage allowance is provided to allow for the change in size when a liquid metal cools and solidifies. For aluminum the allowance is 1.04 percent. Find the shrinkage allowance for the following lengths of aluminum, correct to the nearest tenth of a millimeter. (Caution: be careful how you express 1.04% as a decimal.)

a.	460 mm	_____	mm
b.	216 mm	_____	mm
c.	350 mm	_____	mm
d.	25 mm	_____	mm
e.	762 mm	_____	mm

10. A drafter is asked to make a circle chart. The idea is to divide a circle into sectors in order to show a graphical representation of a budget. To make the proper divisions, the degrees in the central angle of the sector = the percentage times 360 degrees (the number of degrees in the full circle).

Example: Utilities – 16%

Solution: 16% of 360° = 57.6°



Complete the following table:

Budget Item	Percentage	Degrees in Central Angle
Food	21%	
Housing	25%	
Clothing	15%	
Transportation	12%	
Medical exp.	7%	
Recreation	10%	
Savings	6%	
Miscellaneous	4%	

11. The union worked out a new contract with the American Widget Company, calling for a 7.5% increase in pay for all workers for the first year, a 6% increase the second year, and a 4.5% increase the third year. Figure out the final hourly pay rates for these workers. (Hint: remember that the second and third increases are based on the previously raised pay.) Round off each amount to the nearest cent.

	Current Rate	With 7.5% Increase	With 6% Increase	With 4.5% Increase	
a. T. Piper	$3.49				
b. J. Sanchez	3.78				
c. A. Resnick	4.05				
d. M. Robb	5.75				
e. J. Jackson	4.82				

12. a. Make a new column on the above chart. Label it "Total Increase." For this column, work out for each employee the *percent* of increase when the third year's pay is compared with the first year's pay. (Give the percent correct to one decimal place.)

    b. How do you account for the fact that the percents do not equal 7.5 + 6 + 4.5?

    c. How do you account for the fact that the percents vary slightly from each other?

13. Plants may be grown in sand provided they are fed all the chemical elements they need. Below is a formula worked out by the Dutch scientist Abram Steiner for this purpose.

	% Concentration as Dry Fertilizer	kg needed
Nitrogen	8.0%	
Phosphorus	2.2	
Potassium	13.3	
Calcium	8.0	
Magnesium	2.0	
Sulphur	5.7	
Iron	0.13	
Manganese	0.02	
Boron	0.02	
Zinc	0.018	
Copper	0.004	
Molybdenum	0.004	

If a company wished to prepare 2 metric tons (2000 kg) of the Steiner formula, show how many kilograms (kg) of each element would be needed.

Lesson 2 **Memory**

Objectives:

1. You should be able to store a displayed number in the memory register.
2. You should be able to add to, subtract from, and recall the number in the memory register.
3. You should be able to use the memory register in solving problems.

Information:

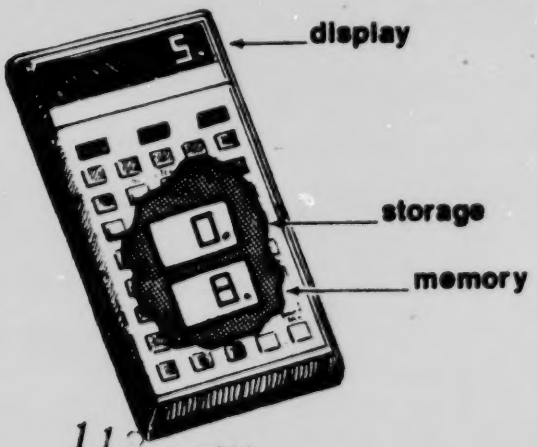
We have dealt so far with two registers on the calculator – the display and the storage registers. Now we will talk about a third register – the memory register.

The memory is a register where we can store a number that we might wish to use again. It can also be used to keep a running total or summation of results as we work out individual figures.

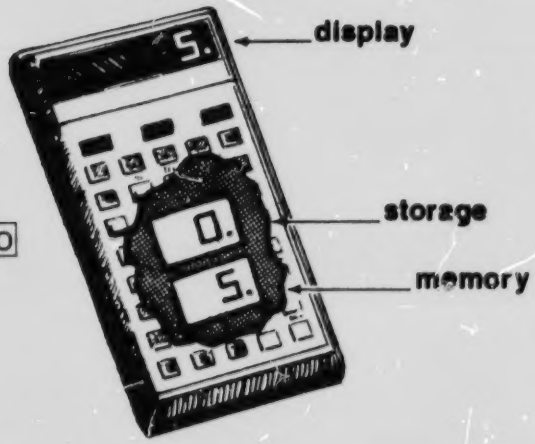
On some calculators, usually the scientific type, the memory key is marked STO for "Store." Pressing this key will *duplicate the number in the display in the memory register.*

When the STO key is used, the principle of destructive read-in is followed in the memory register. Therefore, there is no need to worry about what number might be occupying the memory register when the display number is put into memory with the STO key.

Example: The calculator has a 5 in the display and an 8 in memory. Duplicate the 5 in memory with the STO key.



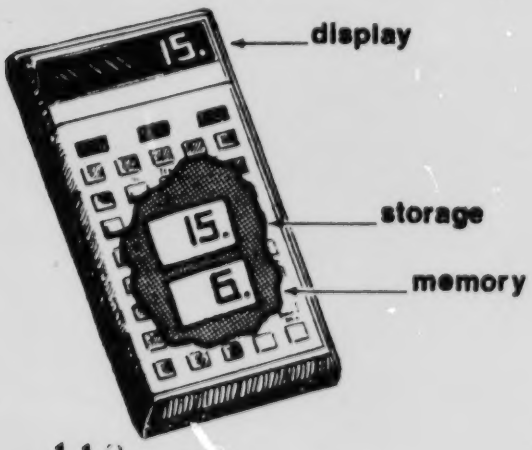
Step 1 – Press [STO]



Notice that putting the 5 into memory did not affect either the display or the storage in any way.

Many calculators have a memory register but do not have a [STO] key. These calculators use the key [M+], "Add to Memory," instead. When this key is pressed, *the number in the display is added to the number in the memory.* If the user wants *only* the number in the display to be in the memory, then he/she must be sure that the contents of the memory register is the number zero. On this type of calculator we find a key [CM], "Clear Memory." This key should be pressed before [M+] to insure that only the number in the display will wind up in the memory.

Example: The calculator has 15 in the display and 6 in memory. Duplicate the 15 in memory, using the [M+] key.



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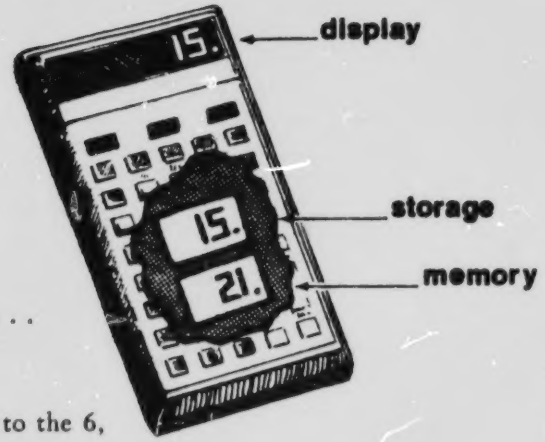
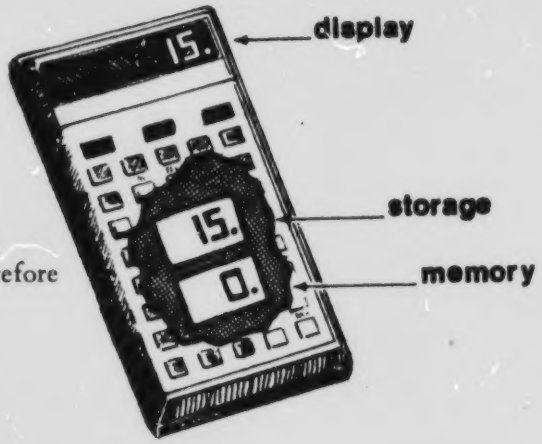
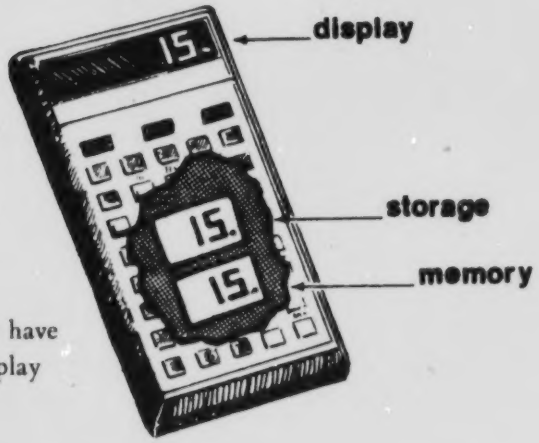


If we press [M+] we will get . . .

Notice that the 15 was added to the 6, resulting in 21 in the memory.



Step 1 – Our first step should therefore be to press [CM] (Clear Memory).



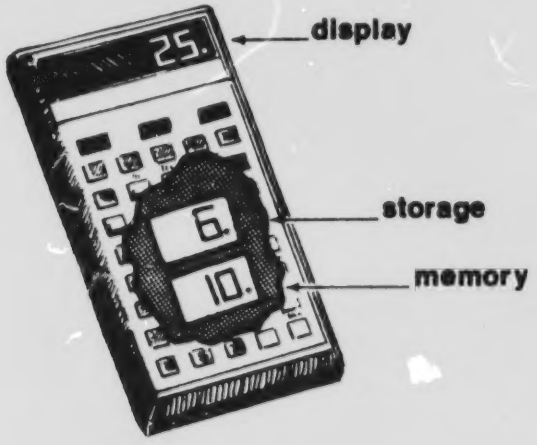
Step 2 – Press [M+]

Now 15 + 0 = 15. Therefore, we have duplicated the number in the display in the memory register.

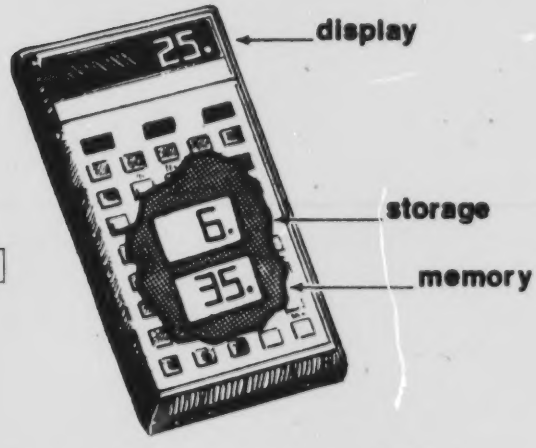
Notice again that pressing the [M+] key did not change either the display or the storage.

The previous explanation showed how to duplicate a number in the display in the memory register. The [M+] key can also be used to *add the number in the display to the number in the memory.*

Example: Add 25 in the display to the 10 in the memory.



Press [M+]



On the keyboard of the calculator with the [M+] key we also find the [M-] key. The [M-] key is used to subtract the number in the display from the number in the memory.

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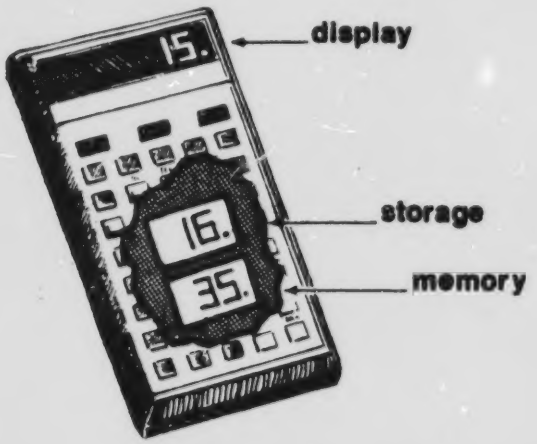
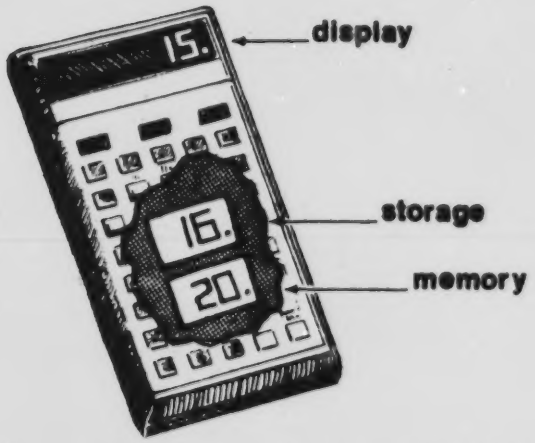
Example: Subtract 15 in the display from the 35 in memory.

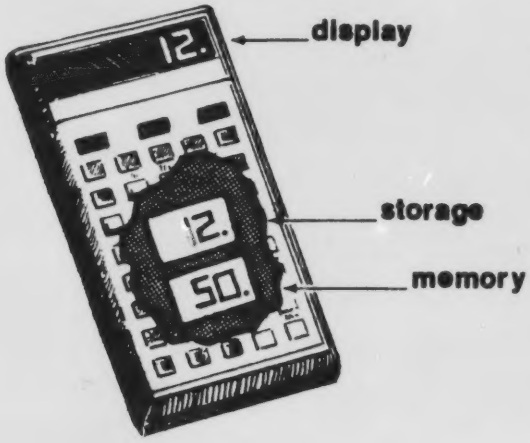


Press M−

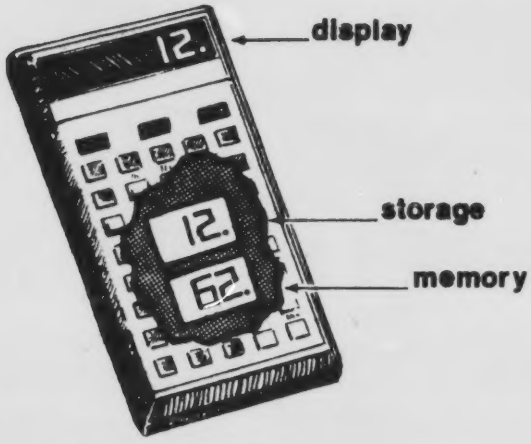


The calculator we mentioned in the beginning of this lesson uses the STO key to duplicate the number in the display in the memory. There was no need to be concerned with the number already in memory because of the destructive read-in principle. In order to *add* the number in the display to the number in memory on this calculator, the user must press the sum key, Σ . The symbol on this key is the Greek capital letter sigma. It is used in mathematics to indicate the sum of a series of terms.

Example: Add the 12 in the display to the 50 in the memory.



Press [Σ]



Most calculators with [STO] and [Σ] keys do not have a direct way to subtract a number from memory. As you learn more about your calculator you will learn how to do this.

When we are using the memory [M] register as a place to keep a number or numbers, we should be careful with our first entry. We must be sure that it is being added to zero. On the calculator with the keys [CM] and [M+] we should press [CM] Clear Memory, before starting our summation. On the calculator with the keys [STO] and [Σ], we simply *make our first entry with the* [STO] *key;* this will erase any number previously stored.

You will recall that the [CM], [M+], [STO], and [Σ] keys do not affect the display or the storage registers.

One more point: the [C], Clear key, does not clear the memory register. You must use the [CM] key. If you should forget to press the [CM] key before starting a new problem, you can press it *at any time during the problem before you need to use the memory*, and all will be well. Also, while working a problem, if you have already used what is in memory, you can cancel the memory and put a new number in it.

Turning the switch off will, of course, clear every number in the calculator.

Machines with [M] keys have some indication on the display that some number is stored in the memory.

Example: Find the total cost of the following items:

a. 3 pens @ $.29
b. 6 pencils @ $.09
c. 2 notebooks @ $1.39
d. 5 dividers @ $.28
e. 2 packages paper @ $.89
f. 25 folders @ $.07

ENTRY	DISPLAY	STORAGE	MEMORY
[C]	0.	0.	?
[CM]	0.	0.	0.
3	3.	0.	0.
[X]	3.	3.	0.
.29	0.29	3.	0.
[=]	0.87	0.29*	0.
[M+]	0.87**	0.29	0.87
6	6.	0.29	0.87
[X]	6.	6.	0.87
.09	0.09	6.	0.87
[=]	0.54	0.09	0.87
[M+]	0.54	0.09	1.41
2	2.	0.09	1.41

* This type of machine usually stores the *previously* displayed figure when the [=] key is pressed.
** The display will also indicate in some way that a number is stored in the memory.

ENTRY	DISPLAY	STORAGE	MEMORY
[X]	2.	2.	141
1.39	1.39	2.	1.41
[=]	2.78	1.39	1.41
[M+]	2.78	1.39	4.19
5	5.	1.39	4.19
[X]	5.	5.	4.19
.28	0.28	5.	4.19
[=]	1.4	0.28	4.19
[M+]	1.4	0.28	5.59
2	2.	0.28	5.59
[X]	2.	2.	5.59
.89	0.89	2.	5.59
[=]	1.78	0.89	5.59
[M+]	1.78	0.89	7.37
25	25.	0.89	7.37
[X]	25.	25.	7.37
.07	0.07	25.	7.37
[=]	1.75	0.07	7.37
[M+]	1.75	0.07	9.12

At this point the answer ($9.12) is located in the memory register only. We must be able to move the answer from the memory register to the display register so that we can learn the answer.

In order to duplicate the number in the memory in the display register we press the Recall key, [RCL] , or [RM] (Recall Memory).

Therefore, the last two steps of the problem above would be:

	ENTRY	DISPLAY	STORAGE	MEMORY
Repeated from above	M+	1.75	0.07	9.12
	RM	9.12	0.07	9.12

Notice that recalling the number from memory *does not take it out of memory* – it is still there. We are simply duplicating it in the display.

If the problem above is done on a calculator with the STO and Σ keys, using STO for storing the first result, 0.87, can replace the CM in the second line. Then the Σ would replace all of the M+ 's. Also, the number in the display would be duplicated in the storage whenever the = key is pressed. (See first footnote on page 119.)

Once a number is recalled from memory to the display, it may be further operated on in any way. But remember that, when a number is recalled from the memory register, it *destroys the number in the* display *register.*

Example:

ENTRY	DISPLAY	STORAGE	MEMORY
2	2.	0.	0.
+	2.	2.	0.
5	5.	2.	0.
=	7.	5.	0.
M+	7.	5.	7.
11	11.	5.	7.
+	11.	11.	7.
RM*	7.*	11.	7.
=	18.	7.	7

* Notice how the 7 recalled to the display register, completely eliminates the 11 from this register. (The 11 is still in the storage register at this point.)
STO and RCL will replace M+ and RM above on some calculators.

Problems:

1. Does your calculator have a memory? If yes, then how many memory registers are there? Is there an indicator in the display that tells you if a nonzero number is in the memory?

2. Show how you clear (set equal to zero) the memory register(s) in your calculator.

3. Show how you duplicate the number in the display register in the memory register.

4. Show how you add the number in the display register to the number in the memory register. Show how you subtract a displayed number from the memory register.

5. Show how you can duplicate the number in the memory register in the display register.

6. What effect does putting a number into memory have on the displayed number? on the number in storage?

7. What effect does recalling a number from memory have on the memory register?

8. What effect does recalling a number from memory have on the number in the display?

9. What precaution should you take before starting to use the memory register?

10. The mass (weight) of several items is listed below. Using the indicated number of each item to be delivered, find the total mass of the entire order.

Number of Items	Mass of Each Item	Subtotals (Mass)
25	60 grams	a.
14	25 grams	b.
135	11 grams	c.
73	215 grams	d.
47	97 grams	e.
	Total Mass	f.

11. Simplify:*

a. $\frac{2653 + 5524}{152 + 329}$ = _____

* (1) Perform the operation in the denominator.
(2) Place the result of (1) in memory.
(3) Perform the operation in the numerator.
(4) Divide the result of (3) by the number in memory.
It is not necessary to clear your calculator between any of the above steps.

b. $\frac{3011.9 + 3095.6}{205.1 - 117.6} =$ _____

c. $\frac{29.5776 \cdot 16}{6.4548 - 6.3972} =$ _____

12. Figure the weekly gross pay for the following workers. All hours over 40 are paid for at time and one-half. Hint: Figure regular pay and store it in memory. Figure overtime pay (if any) and add it to regular pay by using recall key.

Employee	Wage/ Hour	Hours	Regular Pay	Overtime Pay	Gross Pay
a. Arnold, G.	$4.00	38			
b. Buttons, R.	$5.25	40			
c. Cohen, M.	$3.85	37			
d. Davis, W.	$4.25	55			
e. Gonzales, W.	$4.50	60			
f. Martelli, G.	$4.65	35			
g. Novak, J.	$6.00	41			
h. Radon, H.	$8.45	43			
i. Trotsky, I.	$7.00	42½			
j. Williams, H.	$3.45	41½			

13. Compute the weekly gross pay for the following workers, using the same procedure as above.

Employee	Wage/ Hour	Hours	Regular Pay	Overtime Pay	Gross Pay
a. Able, G.	$4.00	38½			
b. Bielski, P.	$3.25	39½			
c. Clark, R.	$5.50	40			
d. Daniels, W.	$6.85	40			
e. Garcio, F.	$3.50	41			
f. Hernandez, J.	$4.05	45½			
g. Irving, A.	$6.25	46¾			
h. Navaro, W.	$3.65	47			
i. Steinman, G.	$4.96	29½			
j. Turner, F.	$3.25	49			

14. Fred Frugal keeps a running tally of his purchases when he shops, and thus can check up on the checkout cashier. Find his total for the following purchases:

	2 @	$.49
	3 @	.55
		1.09
		2.27
	2 @	.69
12 @	3 for	1.00
		.68
	Total	

15. Fred used his calculator to figure out the better buy in his favorite brand of marmalade (the store did not mark "unit" prices). The 12 oz. size cost .79 and the 8 oz. size cost .59. Which was cheaper? _____

16. H. Rodd kept track of his mileage on a trip that covered 962 miles. These were his gas purchases:

7.3	gallons
12.5	"
13.0	"
6.5	"
10.6	"

Without writing down any figures, compute his gas mileage. _____ mpg

17. At the V.I.P. Community College, grades ranged from 0 (failure) to 4 (outstanding). For the year's courses, Charlene had

3	credits	at 0	points
6	credits	at 2	points
15	credits	at 3	points
6	credits	at 4	points
30	credits		

What was her grade-point average for the year? _____

18. Al stockholder held stocks paying the following annual dividends:

200	shares	Public Service paying $1.96 per share
450	"	General Products paying $ .87 per share
300	"	LMX Corp. paying $1.04 per share
94	"	RSVP Industries paying $2.25 per share

What was Mr. Stockholder's annual income from his holdings? _____

19. Widow McGuire inherited a portfolio of bonds when her husband died. These were the amounts and annual interest rates:

$5,000 of Erie Electric Co. paying 4.75%
10,000 of Otter Tail Power Co. paying 6.85%
5,000 of Standard Steel paying 8.375%
15,000 of PR Publishing paying 8.15%

How much annual interest could Mrs. Mc Guire count on? _____

20. Bay Area Gas and Electric charges for electricity at the following rates:

First 20	kilowatt-hours,	$1.95
Next 40	" "	@ 5.2 cents
Next 40	" "	@ 4.4 cents
Above 100	" "	@ 3.75 cents

Arnold Hammer received an electric bill showing a total usage of 116 kilowatt-hours for the month. He wished to check the amount billed him. Using the above schedule, find out how much the bill should have been. _____

21. L'Eclair Bakery had an order from the Tiffany Restaurant for:

9 doz.	hard rolls	@ $1.02/doz.	_____
5 doz.	Danish	@ 1.95/doz.	_____
3	layer cakes	@ 2.05	_____
4	pies	@ 1.50	_____
2 doz.	pastries	@ 2.85/doz.	_____

Compute the price of the individual items and the total price of the order. _____

22. A cashier counting her cash at the end of the day made up the following list.

14	$20 bills	25	half-dollars
102	$10 bills	113	quarters
17	$ 5 bills	125	dimes
6	$ 2 bills	77	nickels
75	$ 1 bills	119	pennies

How much cash was in the cash register? _____

23. Sue Diggs did gardening and maintenance work for homeowners. Rudolph Harayda hired her to plant a shrub border across the back of his property at a cost of $135. Sue calculated her costs as follows:

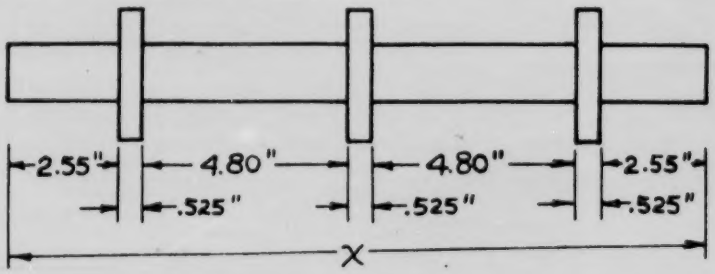
15 shrubs	@	$4.85
Helper – 3 hours	@	3.50/hr.
2 bales peat moss	@	8.50

How much did she make for her own labor and business expenses? _____

24. Go back to page 61 and work problem 30 using constants and memory instead of writing down the total for each item.

25. Work problem 31 on the next page, using memory.

26. Using memory, find the total length of this roller:



Roller

27. The Men's Department of the Dizz Count store received a large shipment of shirts. For competitive reasons the store marked up different types of shirts by different percents.

			Cost Per Style	Markup	Selling Price
Style #3581	18 doz.	$42.00	_____	33%	_____
Style #3093	15 doz.	49.50	_____	25%	_____
Style #1279	5 doz.	62.00	_____	15%	_____
Style # 456	5 doz.	36.20	_____	33%	_____
Style H.K. 359	20 doz.	33.00	_____	50%	_____
Stle H.K. 495	18 doz.	39.50	_____	33%	_____
Style TW 1044	8 doz.	58.00	_____	15%	_____
Style TW 1045	8 doz.	62.50	_____	25%	_____
		Totals	_____		_____

Using constants and memory wherever possible, calculate the cost for each style, then the total cost, then the selling price for each style, and finally the total selling price. Check this last figure by doing the problem horizontally, getting the selling price for each style and then the total selling price.

V – ORDER OF OPERATIONS

Lesson 1 — **Basic Operations**

Objectives:

1. You should be able to state the proper order of operations.
2. You should be able to use the proper order of operations in order to find the correct answer to a given problem.

Information:

In the following expression there are two operations to be performed, addition and multiplication. When simplifying an expression like this, we must decide which operation to perform first. Suppose we try the addition first.

a. $6 + 3 \cdot 4 = 9 \cdot 4 = 36$

Now let's try the multiplication first.

b. $6 + 3 \cdot 4 = 6 + 12 = 18$

One of the answers must be wrong.

In mathematics there is a definite order in which operations must be performed. Some calculators have this order built into the logic of the machine. On the other hand, most do not. Therefore, it is the responsibility of the user to perform the operations in the correct order.

The order of operations is as follows:

A. If the expression does not contain parentheses, then the operations are performed in this order:

   1. Where there are exponents, raise each base to the power indicated by the exponent.
   2. Multiply and divide in the order given, from left to right.*
   3. Add and subtract, from left to right.*

B. Parentheses will be discussed in Lesson 2.

Now let's go back to our example above.

* Remember, however, that multiplication and addition are commutative, i.e., $4 \times 3 = 3 \times 4$.

a. $6 + 3 \cdot 4 = 9 \cdot 4 = 36$

b. $6 + 3 \cdot 4 = 6 + 12 = 18$

Which answer is correct? Now you know.

Example: $2 + 3 + \underline{9 \div 3} = ?$

Step 1 $2 + 3 + 3 =$

Step 2 $5 + 3 = 8$

Example: $76 + 3 \cdot 4 \div 2 - \underline{7^2} - 10 \cdot 3 = ?$

Step 1 $76 + \underline{3 \cdot 4} \div 2 - 49 - \underline{10 \cdot 3} =$

Step 2 $76 + \underline{12 \div 2} - 49 - 30 =$

Step 3 $76 + 6 - 49 - 30 =$

Step 4 $= 3$

While simplifying a problem using the proper order of operations, we can often make use of the memory. This saves us from having to record intermediate answers on a piece of paper.

Example: $7^2 + 2 \cdot 5 + 12 \div 4 = ?$

ENTRY	DISPLAY	STORAGE	MEMORY
[C]	0.	0.	?
[CM]	0.	0.	0.
7	7.	0.	0.
[X^2]	49.	0. (or 7.)	0.
[M+]	49.	0. (or 7.)	49.
2	2.	0.	49.
[X]	2.	2.	49.
5	5.	2.	49.
[=]	10.	5.	49.

(Cont. next page)

ENTRY	DISPLAY	STORAGE	MEMORY
M+	10.	5.	59.
12	12.	5.	59.
÷	12.	12.	59.
4	4.	12.	59.
=	3.	4.	59.
M+	3.	4.	62.
RM	62.	4.	62.

Problems

1. Write the proper order of operations where there are no parentheses.

2. Does your calculator have the order of operations built into the logic of the machine? Enter the following as it appears from left to right.

   6 + 3 X 4 =

3. If the answer in the display is 36, then the calculator *does not have* the order of operations built into the logic of the machine.

   If the answer in the display is 18, then the calculator *does have* the order of operations feature.

3. Use the proper order of operations to simplify each of the following. If your calculator has the built-in order of operations, then just enter the numbers and operations as they appear. If your calculator does not have this feature, then it is your responsibility to follow the proper order of operations. Use the memory register (as shown in the example) to help you when needed.

   a. $24 \div 2 + 3 \cdot 2 =$ _____

   b. $400 - 12.5 \cdot 5 + 113.38 =$ _____

   c. $4^5 \cdot 5 + 1 \cdot 2^6 =$ _____

   d. $1135 \div 5 - 5 + 73 =$ _____

   e. $22.5 + 2.5 \cdot 15.3^2 =$ _____

   f. $1632 \div 2^3 + 9^3 =$ _____

g. $3.005 \cdot 33 + 0.835 =$ _____

h. $113 + 113 \cdot 51 \div 3 =$ _____

i. $143^2 - 13^3 =$ _____

j. $3 \cdot 1.5 + 6 \cdot 12 + 1.4 \cdot 5 =$ _____

k. $2 \cdot 89 - 13 + 16 \cdot 8 + 36 \div 5 =$ _____

l. $15 + 41 \cdot 3 - 3 \cdot 13 - 10 =$ _____

m. $12 \times 13 \div 400 \times 7 + 10 =$ _____

n. $200 - 14^2 \div 4 \times 12 \div 3 =$ _____

4. To convert a temperature from Celsius to Fahrenheit, this formula is used:

$$F = \frac{9 \times C}{5} + 32$$

where F = no. of degrees Fahrenheit
C = no. of degrees Celsius

Find the Fahrenheit equivalent of

a. 20°C _____

b. 37°C _____

c. 0°C _____

d. 100°C _____

5. By experiment it has been learned that sound travels throught the air at about 1090 feet per second at 0°C, and that its speed increases by 2 feet per second for each degree above 0°C. To get the velocity of sound at a given temperature, this formula is used:

$$v = {}^\circ C \times 2 + 1090$$

Find v (velocity) at:

a. 20°C _____ feet/sec

b. 27°C _____ feet/sec

c. 0°C _____ feet/sec

6. A truck had a capacity for carrying 13,000 pounds. The owner wished to load it with 15 crates at 275 pounds each, 8 crates at 225 pounds each, 12 crates at 335 pounds each, and 15 crates at 160 pounds each. Could his truck legally carry the load? If yes, how much extra capacity did the truck have? _____

If no, by how much was it overloaded? _____

7. Alonzo Nunez made out a will leaving half of his estate to his wife, $15,000 each to his three children, and $2,500 to an old friend. The balance was to go to the college he had attended in his youth. If the estate was worth $103,600 when he died, how much did the college get?

_____

8. In one month Mr. Botham Lyne deposited two checks for $352.45 into his checking account, and two checks for $347.45. He wrote three checks for cash for $125 each and other checks totaling $684.02. If his checkbook balance at the beginning of the month was $429.90, what was the balance at the end of the month?

_____

V – ORDER OF OPERATIONS

Lesson 2 **Parentheses**

Objectives:

1. You should be able to state the order of operations including the rule concerning parentheses.
2. You should be able to simplify a given expression using the proper order of operations.
3. You should be able to use the parenthesis keys, [(] and [)] , to solve a problem.

Information:

Parentheses are used in mathematical expressions as symbols to denote the grouping of terms together as one quantity. For example, in the expression $25 - (13 + 2)$, the parentheses around $(13 + 2)$ means that the 13 and the 2 should be combined into one quantity (15) and *then* subtracted from the 25. Similarly, the expression $(10+5)^2$ means that the 10 and the 5 must be combined first (into 15) and *then* squared.

Parentheses are also used simply to indicate the operation of multiplication.

Example: $2 \cdot 3$ may be written $2(3)$ or $(2)(3)$

We will now restate the order of operations from the preceding lesson and add the rule concerning parentheses.

The order of operations is as follows:

A. If the expression does not contain parentheses, then the operations are performed in 3 phases.
   1. Raise each base to the power indicated by exponent.
   2. Multiply and divide in order, from left to right.
   3. Add and subtract, from left to right.

B. If the expression contains one or more pairs of parentheses, then *perform all operations within the parentheses first.* The order of the operations *within* the parentheses is dicated by rule A above.

If there are parentheses *within* parentheses, then work from the inner ones out.

The bar that is used to separate the numerator from the denominator in a fraction (division problem) *acts the same as parentheses.* This means that the quantities above and below the bar need to be combined before the division may be performed.

Example 1: $\frac{6(35 + 73.8)}{19 + 5} = ?$

Step 1 – $\frac{6\ (108.8)}{24} =$

Step 2 – $\frac{652.8}{24} =$

Step 3 – 27.2

Example 2: $(15 + 2.7)^2 = ?$

Step 1 – $(17.7)^2 =$

Step 2 – 313.29

Example 3: $16\ [5 + (2 \cdot 47)]$

(The bracket is used to distinguish the outer from the inner parentheses. Work within the inner pair first.)

Step 1 – $16\ (5 + 94) =$

Step 2 $16\ (99) =$

Step 3 – 1584

Example 4: $16\ (20.2 + 4 \cdot 3) = ?$

Step 1 – $16\ (20.2 + 12) =$

Step 2 – $16\ (32.2) =$

Step 3 – 515.2

Notice that all operations within the parentheses are completed first and in the order dictated in rule A. Notice that the parentheses also indicate the operation of multiplication.

Example 5: 22 (16 - 3.9) + 13 (2.4 + 4.7) = ?

Step 1	22 (12.1)	+	13 (7.1) =
Step 2	266.2	+	92.3 =
Step 3	358.5		

Some machines have keys for parentheses, [(] and [)], and brackets, [[] and []]. If your calculator does not have these keys, then it is up to you to follow the proper order of operations. Use the memory or a piece of paper to keep track of the partial results as the problem progresses.

If the calculator does have the parenthesis keys, [(] [)], then the user should carefully plan the attack on each problem. The parenthesis keys are of no use unless pressed *before* the figures!

Example 1: 16 (20.2 + 4 · 3) = ?

ENTRY	DISPLAY	STORAGE	MEMORY*
16	16.	0.	0.
[X]	16.	16.	0.
[(]	0.	0.	16.
4	4.	0.	16.
[X]	4.	4.	16.
3	3.	4.	16.
[+]	12.	3. (or 12.)	16.
20.2	20.2	12.	16.
[)]	32.2	16.	0.
[=]	515.2	32.2 (or 515.2)	0.

Operation to be remembered

* This is not the same memory register as the one activated by pressing [M+] or [STO]

Notice that an operation must be pressed before the opening parenthesis (if there is a number and operation preceding the parenthesis).

If the operation [X] had not been pressed before the opening parenthesis in the example above, the 16 would not have been sent to memory, but destroyed completely.

Upon pressing the opening parenthesis, the number in the display will be sent to memory. The principle of destructive read-in is followed here. Therefore, any number already in this memory will be lost.

The calculator also remembers the *operation* that is pressed before the opening parenthesis. When the closing parenthesis is pressed, the number in memory is moved to storage, after which the pressing of a basic operation or equal sign will cause the *remembered operation* to take place. The result of this calculation is placed in the display.

Example 2: $\frac{5}{12} + \frac{1}{6} + \frac{8}{9} = ?$

Plan of attack

In this problem we will treat each fraction as if it had parentheses around it. We will not use parentheses around the first fraction, however, because we are not adding it to any previous quantity.

	ENTRY	DISPLAY	STORAGE	MEMORY
	5	5.	0.	0.
	÷	5.	5.	0.
	12	12.	5.	0.
Operation to be remembered	+	0.4166666	12.	0.
	(	0.	0.	0.4166666
	1	1.	0.	0.4166666
	÷	1.	1.	0.4166666
	6	6.	1.	0.4166666
Remembered operation takes place. New remembered operation	)	0.1666666	0.4166666	0.
	+	0.5833332	0.1666666	0.
	(	0.	0.	0.5833332

(continued next page)

ENTRY	DISPLAY	STORAGE	MEMORY
8	8.	0.	0.5833332
÷	8.	8.	0.5833332
9	9.	8.	0.5833332
)	0.8888888	0.5833332	0.
=	1.4722220	0.8888888	0.

Remembered operation takes place.

Therefore, $\frac{5}{12} + \frac{1}{6} + \frac{8}{9} = 1.4722220$ (approx.)

Example 3: $\dfrac{5+7}{\sqrt{15^2 - 209}} = ?$

Plan of attack

We will not use parentheses for our calculation in the numerator, because there is no previous quantity. We will use ÷ as our operation to be remembered. We will treat the denominator as a single quantity, as if it were in parentheses. Therefore we will use the parenthesis keys.

ENTRY	DISPLAY	STORAGE	MEMORY
5	5.	0.	0.
+	5.	5.	0.
7.	7.	5.	0.
÷	12.	7.	0.
(	0.	0.	12.
15	15.	0.	12.
x^2	225.	0.	12.
-	225.	225.	12.
209	209.	225.	12.
=	16.	209.	12.
$\sqrt{x}$	4.	209.	12.
)	4.	12.	0.
=	3.	4.	0.

Operation to be remembered

$\sqrt{x}$ only works on displayed number. Therefore = was needed.

Remembered operation takes place.

Therefore, $\frac{5 + 7}{\sqrt{15^2 - 209}} = 3$

Example 4: $(6 + 7) \cdot (9 + 46) = ?$

(Not really necessary; parentheses indicate multiplication)

Plan of attack

The parentheses in this problem indicate that the additions are to take place before the multiplication (since we work inside parentheses first). Knowing this fact, the user may proceed by adding the 6 and the 7 without actually entering the parentheses. The multiplication will be the remembered operation, and parentheses will be used around (9 + 46).

ENTRY	DISPLAY	STORAGE	MEMORY
6	6.	0.	0.
[+]	6.	6.	0.
7	7.	6.	0.
[×] ← Operation to be remembered.	13.	7.	0.
[(]	0.	0.	13.
9	9.	0.	13.
[+]	9.	9.	13.
46	46.	9.	13.
[)]	55.	13.	0.
[=]	715.	55.	0.

Therefore, $(6 + 7) \cdot (9 + 46) = 715$

Example 5: $(26 + 37^2)^2 \div (312^2 - 97{,}269) = ?$

Plan of attack

Our first step will be to follow the proper order of operations within the first parentheses. Next we will square that quantity and press the division key [÷], which will become the remembered operation. We will use parentheses around the second quantity, $(312^2 - 97{,}269)$.

$$(26 + 37^2)^2 \div (312^2 - 97{,}269) = \ ?$$

ENTRY	DISPLAY	STORAGE	MEMORY
37	37.	0.	0.
x^2	1369.	0.	0.
+	1369.	1369.	0.
26	26.	1369.	0.
=	1395.	26.	0.
x^2	1946025.	26.	0.
÷	1946025.	1946025.	0.
(	0.	0.	1946025.
312	312.	0.	1946025.
x^2	97344.	0.	1946025.
-	97344.	97344.	1946025.
97269	97269.	97344.	1946025.
)	75.	1946025.	0.
=	25947.	75.	0.

Operation to be remembered

Remembered operation takes place.

The answer to this problem is therefore 25947.

The calculator that the author was using for the preceding examples has only one memory register for parentheses. We have shown that the pressing of the opening parenthesis key [(] causes the number in the display to be moved into the memory register with the principle of destructive read-in. Let's examine a problem that has parentheses within parentheses. In studying algebra you might come across something like this.

Example 6: $4\,[18 \div 3\,(32 - 5)] = \ ?$

	ENTRY	DISPLAY	STORAGE	MEMORY
	4	4.	0.	0.
	[X]	4.	4.	0.
Outside parenthesis	[(]	0.	0.	4.
	18	18.	0.	4.
	[÷]	18.	18.	4.
	3	3.	18.	4.
	[X]	6.	3.	4.
Inner parenthesis	[(]	0.	0.	6.

Notice how the pressing of the second opening parentheses caused the 6 in the display to replace the 4 in the memory. Hence the problem cannot be solved using this method on this calculator. (It could be done on a calculator with provision for brackets as well as parentheses.) However, the answer can be obtained by letting the user take the responsibility for proper order of operations and use of parentheses.

Example: $4\,[18 \div 3\,(32 - 5)] = ?$

Plan of attack

First we will perform the operation within the inner parentheses, followed by the operations in proper order within the outer parentheses, followed by multiplication by 4. These operations can be performed on any calculator. The next chart shows a simulation table of the operations on an APF Mark 31 calculator.

$4\ [18 \div 3(32 - 5)] = ?$

ENTRY	DISPLAY	STORAGE	MEMORY
32	32.	0.	0.
[-]	32.	32.	0.
5	5.	32.	0.
[=]	27.	5.	0.
[M+]	27.	5.	27.
18	18.	5.	27.
[÷]	18.	18.	27.
3	3.	18.	27.
[X]	6.	3.	27.
[RM]	27.	6.	27.
[=]	162.	6.	27.
[X]	162.	162.	27.
4	4.	162.	27.
[=]	648.	162.	27.

4 X 162 is equal to 162 X 4.

You can see, then, that if your calculator does not have parenthesis keys, it is of little importance. By using the [M] or [STO] memory register and/or a piece of paper, correct answers can surely be obtained. The most important point is the proper mathematical order of operations.

Example: $(6 + 5) + 48 \div (2^3 + 4) = ?$

ENTRY	DISPLAY	STORAGE	MEMORY
6	6.	0.	0.
[+]	6.	6.	0.
5	5.	6.	0.
[=]	11.	11.	0.
[STO]	11.	11.	11.
2	2.	11.	11.
[x^2]	4.	11.	11.
[X]	4.	4.	11.
2	2.	4.	11.
[+]	8.	8.	11.
4	4.	8.	11.
[=]	12.	12.	11.
48	48.	12.	11.
[÷]	48.	48.	11.
12	12.	48.	11.
[+]	4.	4.	11.
[RCL]	11.	4.	11.
[=]	15.	15.	11.

Record on paper (12).

Problems:

1. Parentheses are often used to indicate the operation of _____.

2. State the proper order of operations including the parenthesis rule.

3. Complete the following simulation tables. Assume the calculator you are using has the parenthesis keys.

   a. 16 (5 + 8) = ?

ENTRY	DISPLAY	STORAGE	MEMORY

b. $\frac{2}{5} + \frac{1}{4} + \frac{3}{8} = ?$

ENTRY	DISPLAY	STORAGE	MEMORY

c. $\frac{\sqrt{200 - 136}}{71 - 63} = ?$

ENTRY	DISPLAY	STORAGE	MEMORY

d. $(13 + 29) \cdot (215 + 197) = ?$

ENTRY	DISPLAY	STORAGE	MEMORY

4. Solve each of the following:

a. $217\ (1038 + 476) =$ _____

b. $438\ (97.5 + 23.8) =$ _____

c. $(18 \div 2) + (27 + 18) =$ _____

d. $3\ (14 \div 2)^2 + 137 =$ _____

e. $(348 - 332)^2 \div (8 \div 16) =$ _____

f. * $\frac{7}{9} + \frac{2}{3} + \frac{3}{7} =$ _____

g. * $\frac{4}{17} + \frac{25}{136} + \frac{14}{193} =$ _____

h. $\frac{\sqrt{761 - 136}}{5}$ _____

i. $\frac{143 + 217}{\sqrt{3{,}240{,}000}}$ _____

j. $(3.9 + 21.8)^3 =$ _____

k. $(18.56 + 2.8)^4 =$ _____

l. $4\ (2.56 + 3.87)^3 =$ _____

m. $(2470 \div 26)^2 \div 25 =$ _____

n. $(29 + 84) \cdot (105 + 378) =$ _____

o. $(125 \div 5) + (2{,}397 + 1{,}576) =$ _____

p. $22\ (16 - 3.9) + 13\ (2.4 + 4.7) =$ _____

q. $\frac{5\ (37.2 + 0.375)}{25} =$ _____

r. ** $\frac{3\ (14.2 + 15.9)}{22} =$ _____

5. Solve each of the following. Use a piece of paper for partial results if necessary.

a. $(5.2)^2 + (3.6 \cdot 4.8) =$ _____

b. $35\ (1.05^2 + 2.79^2) =$ _____

c. $9\ [\ 16^2 + (4.2 \cdot 5)\ ] =$ _____

* Round off to the hundredths place.
** Round off to the thousandths place.

d. $14\,[3.1^2 + (12 - 7.8)\,] =$ _____

e. $3\,[\,(14 - 6.2)^2 - (18 - 3.7)] =$ _____

f. $[(14 \cdot 2) + 16] \div 11 =$ _____

g. $[(8974 - 6237) \div 7]^3 =$ _____

h. $\dfrac{\sqrt{29^2 + 5 \cdot 63}}{4 \cdot 17} =$ _____

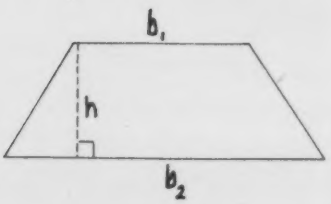
i. $\dfrac{[(896 + 354)^2 \cdot 16]}{\sqrt{(1897 + 1103) \div 4.8}} =$ _____

6. In converting from Fahrenheit temperature to Celsius we can use the formula

$$°C = \frac{°F - 32}{1.8}$$

Complete the table.

°F	°C
a. 32°	
b. 68°	
c. 98.6°	
d. 212°	

7. 

The area of a trapezoid is given by the formula

$$A = .5h\,(b_1 + b_2)$$

where h = the height of the trapezoid,
b_1 = the length of base 1
b_2 = the length of base 2. (The bases are the parallel sides.)

If h = 36 mm, b_1 = 30 mm and b_2 = 80 mm, then A = _____ mm^2.

8. When a loan is paid off in installments, the true annual rate of interest that is being paid can be found by using the formula

$$r = \frac{2YI}{P\,(n + 1)}$$

where Y = number of interest periods per year
I = dollar amount of interest charged
P = principal (amount borrowed)
n = number of installments needed to repay the loan.

Stuart bought a small nylon tent on the installment plan, paying \$80 for the tent plus \$9 interest. He agreed to pay for it in 8 equal monthly payments of \$11.13. Then Y = 12, I = \$9, P = \$80, and n = 8. Find the true annual interest rate.

_____

9. When interest on money is compounded, the compound (final) amount can be calculated by using the following formula.

$$C = P \cdot (1 + r)^n$$

where C = compound amount
P = principal
r = rate of interest and
n = number of interest periods.

If P = \$1,000, r = .03 (3%), and n= 10, then C = \$ _____.

10. Go back to page 97 and solve problem 29 by the above formula.

11. Solve problem 30 on the same page by the above formula.

12. People who buy homes normally pay a certain amount in cash, and take out a mortgage for the rest of the purchase price. In other words, they borrow that amount from a bank, agreeing to pay it back over a period of years, with interest at a certain annual interest rate. (The bank that holds the mortgage has a right to take possession of the house if the payments are not made.)

To calculate the monthly payment for a home mortgage, we can use the formula

$$M = P \left(\frac{r C}{C - 1} \right)$$

where M = monthly payment toward the mortgage
P = principal (amount borrowed)
r = monthly interest rate and
C = compound amount of \$1 over the entire mortgage period.
(The amount for C is obtained from a printed chart. All banks have them.)

If P = \$36,000, r = .005 (monthly rate for 6% annual interest), and C = 4.4649699 (after 25 years), then M = \$ _____.

13. An important concept in business and industry is depreciation – the fall in value of machinery, buildings, etc., as time goes on. Businesses have to set aside funds for depreciation in their accounting procedures. To estimate depreciation by the so-called straight-line method, we are able to use the formula

$$d = \frac{C - f}{n}$$

where d = annual straight-line depreciation
C = original cost
f = estimated scrap or resale value
n = number of years of estimated useful life.

If C = \$4,500, f = \$1,368, and n = 12, then d = \$ _____.

14. If a student had the following grades on 14 tests: 100, 95, 93, 97, 86, 92, 79, 53, 84, 77, 91, 65, 84, and 75, the average grade would be
(Round off to the nearest whole number) _____

15. The coefficient of linear expansion of a solid is the amount a unit length of a solid expands for each degree of rise in temperature. Problems involving linear expansion are solved by the following formula:

$$\text{Coefficient of linear expansion} = \frac{\text{Change in length}}{(\text{Original length}) \times (\text{Change in temperature})}$$

$$\text{Coefficient} = \frac{l_2 - l_1}{l_1 \times (T_2 - T_1)}$$

A piece of brass is 40.000 in. long at 10°C. When heated to 110°C, it is found to be 40.076 in. long. What is the coefficient of linear expansion? _____

16. In a parallel electric circuit with two resistances, the total resistance of the circuit may be found by this formula:

$$R_{TOT} = \frac{R_1 \times R_2}{R_1 + R_2}$$

If R_1 = 8 ohms and R_2 = 15 ohms, the total resistance is _____ ohms.
(Round off to the nearest tenth.)

17. In a parallel electric circuit with any number of resistances, the formula for finding the total resistance of the circuit is

$$R_{TOT} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots}$$

If there are three resistances in a circuit, of 10, 5, and 2 ohms respectively, find the total resistance. _____ ohms.

18. Find the total resistance of a parallel circuit where the individual resistances are 12, 15, and 18 ohms. (Round off to the nearest tenth.) _____ ohms.

19. James Carter decided to add additional insulation to his attic floor. There were three separate levels to the floor, which measured 18′ × 18′, 18′ × 11′, and 13′ × 11′. He decided on a type of insulation where each roll covered 48 sq. ft. How many rolls did he have to buy? _____

20. Richard Nickson was planning to paint his living and dining rooms. To estimate his paint needs, he measured his walls. Two were 16 feet long, two were 14 feet, two were 13 feet, and two were 12 feet. All were 8 feet high. He did not allow for windows and doors (which is the customary practice). At a coverage of 450 square feet per gallon, how many gallons would Mr. Nickson need for *two* coats of paint.?
_____ gal.

21. The taper on a piece of work to be machined is the difference between the diameters at the ends of the work. The taper per inch of length can be found by this formula:

$$T.P.I. = \frac{D - d}{L}$$

where D = large diameter
d = small diameter
L = length of tapered part in inches

Find the taper per inch of a piece of work 4.0 inches long if D = .625" and d = .515".

VI – SPECIAL KEYS

Lesson 1 — **π Key**

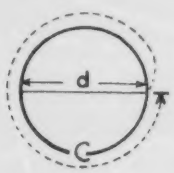
Objectives:

1. You should be able to state the given definition for pi (π).
2. You should be able to use the [π] key on your calculator.

Information:

In every circle, if you divide the circumference by the diameter, you get the same answer. It is roughly $\frac{22}{7}$. This number has been given the name π, (pi, a Greek letter).

We can define π as the ratio of the length of the circumference to the length of the diameter of a circle.



$$\pi = \frac{C}{d}$$

The very rough approximation for π, $\frac{22}{7}$, comes to 3.142857143... if you divide out the fraction. (Check it.)

Archimedes, who lived before the birth of Christ, is said to have known that π was somewhere between 220/70 and 223/71. (Try these on your calculator and see what you get.)

The common fraction 355/113 gives us the first seven of the digits in π. (335/113 = 3.14159292...)
7 digits

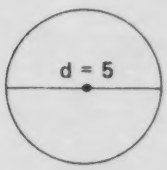
The number π is called an irrational number. This means it never ends nor does it repeat a pattern of digits. It cannot be obtained by any common fraction. Therefore, all the fractions mentioned so far have given us only approximations.

In higher mathematics π can be calculated. The first 11 digits are given below. This is more than most calculators display.

$$\pi = 3.1415926536\ldots$$

The number π is found in all mathematical formulas involving circles, and in many other formulas as well.

Example: Find the circumference of the given circle.



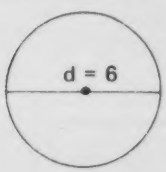
$C = \pi d$ or $2\pi r$

where C = circumference
d = diameter
r = radius

ENTRY	DISPLAY	STORAGE
[π]	3.1415926	0.
[X]	3.1415926	3.1415926
5	5.	3.1415926
[=]	15.707963	15.707963

Therefore, the circumference is 15.707963. Of course, since the original measurement was 5, this answer "pretends" to an accuracy that it really doesn't have (see page 75). We should round off the circumference to 15.7.

Example: Find the area of the given circle.



$A = \pi r^2 = \pi (\frac{d}{2})^2$

where A = area of the circle
d = diameter
$r = \frac{d}{2}$ = radius

ENTRY	DISPLAY	STORAGE
3	3.	0.
[X²]	9.	0.
[X]	9.	9.
[π]	3.1415926	9.
[=]	28.274333	28.274333

Therefore, the area is 28.27433 or 28.3 square units.

Problems:

For the problems below, use the $\boxed{\pi}$ key if your calculator has one; if not, use 3.1416 for π.

1. State the given definition for pi (π) .

2. a. Which of the given common fractions is closest to π?

   (1) $\frac{22}{7}$

   (2) $\frac{220}{70}$

   (3) $\frac{223}{71}$

   (4) $\frac{355}{113}$

   b. Find the difference between π (using the number of digits your calculator is capable of) and the above common fractions.

3. Find the approximate value for each expression.

   a. $\frac{1}{2}\pi$ =

   b. π =

   c. $\frac{3}{2}\pi$ =

   d. 2π =

   e. $3\pi\ (1.05^2 + 2.79^2)$

   f. $4.37\pi\ (2.01 + 0.05)^2$ =

4. The circumference of a circle is given by the formulas

$$C = \pi d \quad \text{or} \quad C = 2\pi r$$

where C = circumference
d = diameter
r = radius

Complete the following. Round off to the nearest tenth.

a.

d	C
24 mm	mm
36 mm	mm
4.1 cm	cm
2.04 cm	cm
165 cm	cm

b.

r	C
36 mm	mm
38.1 mm	mm
20 mm	mm
12.3 cm	cm
7.05 cm	cm

5. The area of a circle can be found by using the formula

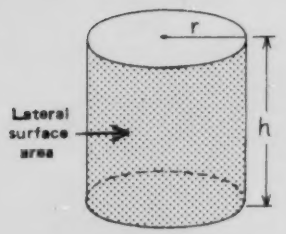
$$A = \pi r^2$$

where r = the radius of the circle.

Complete the table.
Round off to the nearest hundredth.

r	A
a. 35.60 in	
b. 0.52 cm	
c. 18.50 m	
d. 7.84 ft.	
e. 195.01 m	

6.



Right Circular Cylinder

The lateral surface area (for example, the area covered by the label on a soup can) of a right circular cylinder is given by the formula

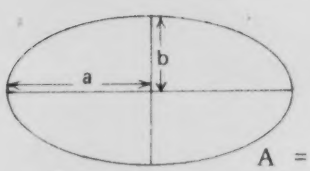
$$A = 2\pi rh$$

where r = radius of the base of the cylinder
h = height of the cylinder

Complete the following table. Round off to the nearest hundredth.

Radius	Height	Lateral Surface Area
a. 5 mm	12 mm	mm^2
b. 5 mm	4 mm	mm^2
c. 3.2 cm	3.7 cm	cm^2
d. 50 mm	18.7 mm	mm^2
e. 73.25 m	48.79 m	m^2

7.



ELLIPSE

$$A = \pi ab$$

The area of an ellipse is found by using the formula

where a = distance from the center to the ellipse along the major axis
b = distance from the center to the ellipse along the minor axis

If b = 150 ft. and a = 325 ft., then A = _____ sq. ft.
(Round off to the nearest tenth)

8. The circumference of an ellipse is found by using the formula

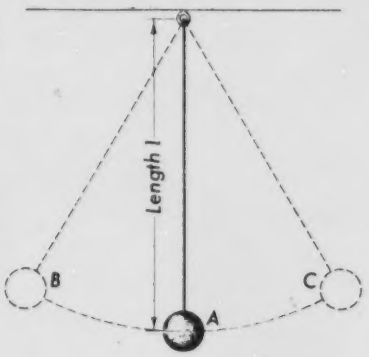
$$C = 2\pi \sqrt{\tfrac{1}{2}(a^2 + b^2)}$$

If a = 150 ft., and b = 325 ft., then C = ____ ft. (Round off to the nearest tenth.)

9. The period of a pendulum is the time required for a complete swing (from B to C and back again). It is given by the formula

$$T = 2\pi \sqrt{\frac{l}{g}}$$

where l = length of the pendulum
g = acceleration due to gravity.



If l = 10 ft. and g = 32 ft/sec^2, then T = _____ sec. (Round off to the nearest tenth.)

10. The speed of a body being whirled in a horizontal circle is given by the formula

$$V = 2\pi rf$$

where r = radius of the circle
f = speed of the body in rev/sec.

If r = 6 ft., and f = 3 revolutions per sec., then V = ____ ft./sec. (Round off to the nearest tenth.)

11. Any coil of wire carrying a current will develop a magnetic field around it. The strength of the field can be found by the formula

$$\text{field strength} = \frac{2\pi IN}{L}$$

where I = amperes of current
N = number of turns of wire
L = length of the magnetic lines inside the coil in centimetres.

If I = 3 amps, N = 200 turns, and L = 10 cm, then field strength = _____ ampere-turns. (Round off to the nearest tenth.)

12. The horsepower of an electric motor can be found by the formula

$$\text{h.p.} = \frac{2\pi T \cdot (\text{r.p.m.})}{33{,}000}$$

where T = torque
r.p.m. = revolutions per minute.

If T = 5.65 lb-ft and r.p.m. = 1250, then h.p. = _____ horsepower. (Round off to the nearest tenth.)

13. The cutting speed of a drill can be found by using the formula

$$S = \frac{\pi d \cdot (r.p.m.)}{12}$$

where S = cutting speed
d = diameter of the drill bit in feet
r.p.m. = revolutions per minute.

If $d = 0.0208$ (approx. ¼ inch) and r.p.m. = 300, then S = _____ ft./min. (Round off to the nearest hundredth.)

14. The formula for brake horsepower is

$$b.h.p. = \frac{2\pi L \cdot (r.p.m.) \cdot F}{33{,}000}$$

where L = length of the brake arm in feet
F = force exerted
r.p.m. = revolutions per minute of the pulley.

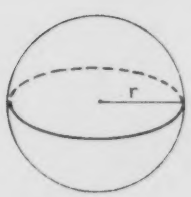
If $L = 2$ feet, r.p.m. = 1800, and $F = 15$ lb., then b.h.p. = _____ h.p. (Round off to the nearest tenth.)

15. The surface speed of a pulley is found by using the formula

$$S = 2\pi rn$$

where S = surface speed in inches per minute
r = radius of the pulley in inches
n = number of revolutions per minute.

If $r = 8$ inches, and $n = 1200$ r.p.m., then S = _____ inches per minute. (Round off to the nearest tenth.)

16. 

SPHERE

The volume of a sphere can be found by using the formula

$$V = \frac{4}{3}\pi r^3$$

where r = the radius.

If $r = 25.7$ cm, then V = _____ cm^3. (Round off to the nearest tenth.).

17. The surface area of a sphere can be found by using the formula

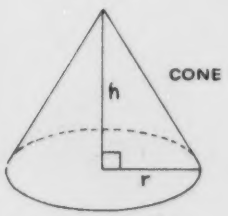
$$S = 4\pi r^2$$

If r = 4.68 cm, then S = _____ $cm.^2$. (Round off to the nearest hundredth).

18. The volume of a cone can be found by using the formula

$$V = \frac{1}{3}\pi r^2 h$$

where r = radius of the base
h = the height of the cone.



If r = 27.25 m and h = 20.31 m, then V = _____ m^3. Round off to the nearest hundredth).

19. The surface area of a cone can be found by using the formula

$$S = \pi r^2 + \pi r \sqrt{r^2 + h^2}$$

If r = 4.2 cm and h = 15.2 cm, then S = ____ cm^2. (Round off to the nearest tenth.)

20. Which is larger, a square with a side 16 inches long, or a circle with a diameter of 18 inches? (Hint: the formula for the area of a circle will be found in problem 5.)

_____

VI – SPECIAL KEYS

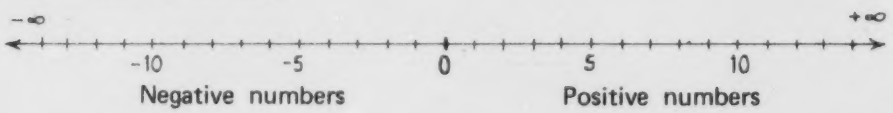
Lesson 2 — **Negative Numbers and Sign-Change Key** [+/-]

Objectives:

1. You should be able to state the meaning of a negative number.
2. You should be able to state the meaning of the word "minus."
3. You should be able to find the negative indication in your calculator's display.
4. You should be able to add, subtract, multiply, and divide negative numbers on your calculator.
5. You should be able to use the sign-change key, [+/-] , on your calculator to reduce the number of operations when solving certain types of problems.

Information:

A **negative number** is a number that is less than zero. If we represent all numbers as points along a line, the negative numbers are located to the left of zero on the line.

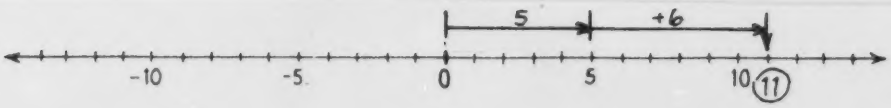


Pretend you are standing in a room wearing only a bathing suit and you have to choose between two exits. Behind one door the temperature is a nice warm 84°F (or approximately 29°C). Behind the other door the temperature is a bitter cold −84°F (or approximately −64°C). Can you see the significance of the negative sign? It should be very clear that the numbers 84 and −84 are two entirely different numbers, just as 6 is different from 10.

Addition and subtraction are often explained as *directed movement along the number line.*

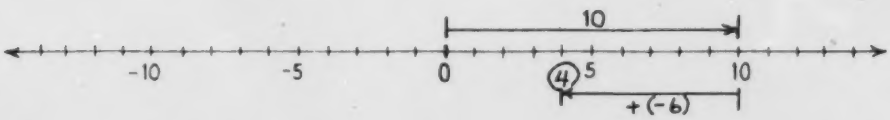
Addition of a *positive* number is viewed as movement *to the right.*

Example: 5 + 6 = 11



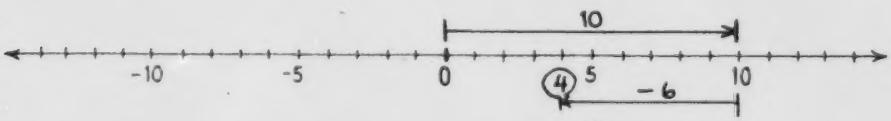
Addition of a *negative* number is viewed as movement *to the left* along the line.

Example: 10 + (-6) = 4



Subtraction of a *positive* number is also viewed as movement *to the left.* When we read a subtraction problem we often use the word "minus" to indicate the operation of subtraction.

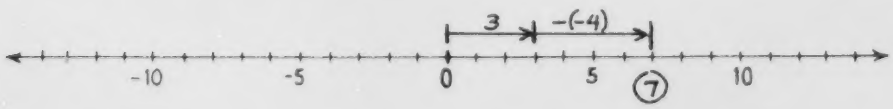
Example: 10 - 6 = 4



Notice that addition of a negative number and subtraction of a positive number are both viewed as movement to the left.

Subtraction of a *negative* number is viewed as movement *to the right.*

Example: 3 -(-4) = 7



To help you remember this, you might think of *minus a negative* as "don't go to the left". Therefore, you go in the other direction, which is
(negative) (minus)

to the right. Also we might say that two negatives yield a positive, just as in an English sentence.

Example:

I did *not* say I would *not* go.
means
I will likely go.

Calculators differ greatly in their handling of negative numbers. On many calculators the negative sign is indicated at the extreme left of the display. On some it is immediately in front of the number.

In calculating you may wish to change the sign of the number in the display. The calculators being used by the author use the function key, [+/-] , to change the sign of any number in the display (regardless of how it got there). With other popular models, the sign in the display can be changed by first pressing the minus (subtraction) key, [-] , and then pressing the equal sign twice. Check your owner's manual if there is any question about your own calculator.

Once you get a negative number on the display, it can be used for any calculation as if it was a positive number. It can also be put into memory.

Examples: Addition and subtraction with negative numbers.

Note: many calculators will require entries different from those shown below. On some it is possible to enter a negative number by merely pressing the [-] key before pressing the number.

a. 1491 + (-43) = ?

ENTRY	DISPLAY	STORAGE
1491	1491.	0.
[+]	1491.	1491.
43	43.	1491.
[+/-]	-43.	1491.
[=]	1448.	1448.

Notice that the 43 had to be entered into the display before the sign was changed to negative.

b. 1491 - 43 = ?

ENTRY	DISPLAY	STORAGE
1491	1491.	0.
[-]	1491.	1491.
43	43.	1491.
[=]	1448.	1448.

Notice that (a) and (b) gave the same result. This illustrates the fact that addition of a negative number and subtraction of the same positive number give the same result.

$$1491 + (-43) = 1491 - 43$$

c. -25 - 16 = - 25 + (-16) = ?

ENTRY	DISPLAY	STORAGE
25	25.	0.
[+/-]	- 25.	0.
[-]	- 25.	- 25.
16	16.	- 25.
[=]	- 41.	- 41.

d. 119 - (-42) = 119 + 42 = ?

minus negative

ENTRY	DISPLAY	STORAGE
119	119.	0.
[-]	119.	119.
42	42.	119.
[+/-]	-42.	119.
[=]	161.	161.

or simply:

ENTRY	DISPLAY	STORAGE
119	119.	0.
[+]	119.	119.
42	42.	119.
[=]	161.	161.

e. $9 - 7 + (-6) - 17 = ?$

ENTRY	DISPLAY	STORAGE
9	9.	0.
[-]	9.	9.
7	7.	9.
[-]	2.	2.
6	6.	2.
[-]	-4.	-4.
17	17.	-4.
[=]	-21.	-21.

Notice that it was easier to simply view plus negative 6 as minus positive 6.

When we are working with positive and negative numbers in multiplication and division, we should be aware of some basic rules regarding the sign of the results. (The calculator will automatically do this for you).

Examples: $5 \cdot 2 = 10$
$10 \div 2 = 5$

a. positive X or ÷ by positive = positive

$$-3 \cdot (-6) = 18$$
$$-6 \div (-3) = 2$$

b. negative X or ÷ by negative = positive

$$6 \cdot (-7) = -42$$
$$24 \div (-8) = -3$$

c. positive X or ÷ by negative = negative

$$-2 \cdot 7 = -14$$
$$-16 \div 2 = -8$$

d. negative X or ÷ by positive = negative

In chain multiplication and division problems, if there is an *odd* number of minus signs, the answer will be negative. If there is an *even* number of minus signs, the answer will be positive.

Examples: Multiplication and division with positive and negative numbers.

a. $57 \cdot (-137) = ?$

ENTRY	DISPLAY	STORAGE
57	57.	0.
[X]	57.	57.
137	137.	57.
[+/-]	-137.	57.
[=]	-7809.	-7809.

b. $-2520 \cdot 33 = \ ?$

ENTRY	DISPLAY	STORAGE
2520	2520.	0.
[+/-]	-2520	0.
[X]	-2520.	-2520.
33	33.	-2520.
[=]	-83160.	-83160.

c. $-1783 \cdot (-876) = \ ?$

ENTRY	DISPLAY	STORAGE
1783	1783	0.
[+/-]	-1783.	0.
[X]	-1783.	-1783.
876	876.	-1783.
[+/-]	-876.	-1783.
[=]	1561908.	1561908.

The user who realizes that *a negative times a negative is a positive* could have left out the sign changes. The final answer would be the same.

d. $-41080 \div 65 = \ ?$

ENTRY	DISPLAY	STORAGE
41080	41080	0.
[+/-]	-41080	0.
[÷]	-41080.	-41080.
65	65.	-41080.
[=]	-632.	-632.

e. $-13818 \div (-987) = ?$

ENTRY	DISPLAY	STORAGE
13818	13818.	0.
[+/-]	-13818.	0.
[÷]	-13818.	-13818.
987	987.	-13818.
[+/-]	-987.	-13818.
[=]	14.	14.

f. $-152 \cdot (-431) \cdot (-64) \div 608 = ?$

ENTRY	DISPLAY	STORAGE
152	152.	0.
[+/-]	-152.	0.
[X]	-152.	-152.
431	431.	-152.
[+/-]	-431.	-152.
[X]	65512.	65512.
64	64.	65512.
[+/-]	-64.	65512.
[÷]	-4192768	-4192768.
608	608.	-4192768.
[=]	-6896.	- 6896.

g. $5 + (-7)^2 = ?$

Notice that the number negative seven is being squared. Use proper order of operations.

ENTRY	DISPLAY	STORAGE
7	7.	0.
[+/-]	-7.	0.
[X²]	49.	0.
[+]	49.	49.
5	5.	49.
[=]	54.	54.

Suppose we had the number 7532 in the display (after a previous calculation), and we wished to subtract it from the number 10000. Subtraction is not commutative; therefore if we press the subtraction key and enter 10000 followed by the equal sign, we will have an answer *with the wrong sign.*

$$(10000 - 7532 \neq 7532 - 10000)$$

(What we want) $(2468 \neq -2468)$ (What we have)

However, we can eliminate this problem by *making the 7532 in the display negative and then adding 10,000.* We do this by using the [+/-] key.

$$10{,}000 - 7532 = -7532 + 10{,}000 = 2468$$

Example: $5281 - (36)^2 = ?$

Using the proper order of operations, we will have to square the 36 first. This will leave us with 1296 in the display. Up to now we would have had to jot the number down or put it into memory in order to subtract it. Now the [+/-] key will come to the rescue.

ENTRY	DISPLAY	STORAGE
36	36.	0.
[X²]	1296.	0.
[+/-]	-1296.	0.
[+]	-1296.	-1296.
5281	5281.	-1296.
[=]	3985.	3985.

Therefore $5281 - (36)^2 = -(36)^2 + 5281 = 3985$

Another solution to the above problem would be to subtract in reverse order and then change the sign of the answer yourself.

Problems:

1. State the meaning of the term "negative number."
2. What does the word "minus" mean?
3. How does your calculator indicate that the number in the display is negative?
4. How do you change the sign of a number in the display on your calculator?
5. Does the [+/-] key change the sign of the number in the storage register? In the memory register?
6. Complete the following simulation table. Use the [+/-] key if your calculator has one. If not, show the procedure on your own calculator, and see if you can supply the figures for the last column as well.

$5 + (-3) = ?$

ENTRY	DISPLAY	STORAGE

7. Complete the following simulation table. Use the [+/-] key or the system on your own calculator.

$81 - (7^2 + 10) = ?$

ENTRY	DISPLAY	STORAGE

8. Are the expressions 8 - (-10) and 8 + 10 equal? Explain.

9. Perform the indicated operations.

   a. 13.25 + (-2.11) =

   b. 13.25 - 2.11 =

   c. 4802.051 + (-387.275) =

   d. 4802.051 - 387.275 =

   e. 0.053 + (-0.047) =

   f. 0.053 - 0.047 =

   g. 549 + (-783) =

   h. 549 - 783 =

   i. 46.703 + (-328.615) =

   j 46.703 - 328.615 =

10. What generalization can be made after completing problem #9?

11. State the four rules concerning the signs in a multiplication and/or division problem.

12. Perform the indicated operations. (If your calculator is the type that permits you to simply use the [-] key before a negative number, be sure to *clear the display* before starting a problem whose first number is negative.)

    a. $25 \cdot (-33) =$ _____

    b. $-167 \cdot (-381) =$ _____

    c. $98.32 \cdot (-2.86) \cdot (-4.71) =$ _____

    d. $-13.27 \cdot 87.2 =$ _____

    e. $-57.28 \cdot 0.563 =$ _____

    f. $-325 \cdot (-5.61) \div (-25) =$ _____

    g. $-19509.03 \div 3.69 =$ _____

    h. $35.75312 \div (-0.052) =$ _____

    i. $-64 \div (-0.0016) =$ _____

    j. $-5828 \div 4 \cdot (-0.031) =$ _____

    k. $-5828 \div [4 \cdot (-0.031)] =$ _____

    l. $-22976 \div [(-359) \div (-1.6)] =$ _____

13. Perform the indicated operations.

    a. $52.37 + (-48.76) - 37.52 =$ _____

    b. $-68.7 - (-36.3) + (-14.8) =$ _____

    c. $0.576 + (-8.321) + 14.710 =$ _____

    d. $16^2 + (-15)^2 =$ _____

    e. $253 + [12^2 + (-47)] =$ _____

    f. $5184 \div [36^2 - (-144)] =$ _____

    g. $-897 \cdot (-5.92) \div (-0.06) =$ _____

    h. $(1984 + 68) \div (-36) + (-3) =$ _____

    i. $18^2 - (6.3^2 - 4.9^2) =$ _____

j. $27 - [42.8^2 + (-8.037)] =$ _____

k. $896 + (-2.8)^3 + (-4.1)^4 =$ _____

14. In algebra if an equation is in the form

$$ax^2 + bx + c = 0$$

There will be two correct solutions for x.

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

In the equation

$$5x^2 + 9x - 2 = 0$$

$a = 5$, $b = 9$, and $c = -2$.

Find the correct values for x. _____

_____

15. Denise had a checkbook balance of $34.07. She wanted to write checks for three bills, amounting to $31.40, $15.50, and $42.95. She had a birthday check for $75 to deposit. Would there be enough money in the bank to cover the checks? _____
If so, how much would be left in the account? _____
If not, how short would the account be? _____

16. Francis S. Key received a stack of bills on the first of the month and wrote out checks for all of them. The amounts were: $25.67, $44.90, $18.19, $200.50, $65.00, and $8.75. Before writing the checks his bank balance had been $202.79. He had two checks to deposit, for $100 and $296.45. What was his new bank balance?

_____

17. A truck with a capacity of 12,000 pounds has already been loaded with 25 crates @ 148 pounds each and 28 crates @ 168 pounds each. How much capacity is left?

_____

18. While shopping in a variety store, Penny got "carried away" and had in her basket merchandise priced at $4.59, $1.25, $3.98, $4.98, and $7.35. She had only a $20 bill with her. Would she have enough money to pay for all the purchases? _____
If so, how much would be left over? _____
If not, how much short would she be? _____

19. The majority "whip" in the House of Representatives was trying to get enough votes to pass a very important bill. If all members were present, the bill would need 218 votes to pass. His five helpers reported 24, 33, 16, 18, and 31 members definitely committed to the bill. How many more votes would be needed to pass it?

_____

Lesson 3 — **Reciprocal Key** [1/x]

Objectives:

1. You should be able to state the meaning of the reciprocal of a number.
2. You should be able to find the reciprocal of the number in the display by using the reciprocal key [1/x] .

Information:

If x is a given number (not zero), then $\frac{1}{x}$ is the reciprocal of x. It follows that $x \cdot \frac{1}{x} = 1$. In algebra we sometimes express $\frac{1}{x}$ as x^{-1}.

Examples: $\frac{1}{5}$ is the reciprocal of 5; $5 \cdot \frac{1}{5} = 1$

$-\frac{1}{2}$ is the reciprocal of -2; $-2 \cdot (-\frac{1}{2}) = 1$

6 is the reciprocal of $\frac{1}{6}$; $\frac{1}{6} \cdot 6 = 1$

If x is a common fraction such as $\frac{2}{3}$, then the reciprocal is $\frac{1}{\frac{2}{3}} = \frac{3}{2}$.

Examples:

$\frac{4}{5}$ is the reciprocal of $\frac{5}{4}$; $\frac{5}{4} \cdot \frac{4}{5} = 1$

$-\frac{3}{7}$ is the reciprocal of $-\frac{7}{3}$; $-\frac{7}{3} \cdot (-\frac{3}{7}) = 1$

Here are two ways to find the reciprocal of a given number x. (Remember that $\frac{1}{x}$ means $1 \div x$.)

Example: Find the reciprocal of 5.

a.

ENTRY	DISPLAY	STORAGE
1	1.	0.
[÷]	1.	1.
5	5.	1.
[=]	0.2	0.2

The reciprocal of 5 is $\frac{1}{5}$ or 0.2, as expressed in decimal form in the calculator display.

b.

ENTRY	DISPLAY	STORAGE
5	5.	0.
÷	5.	5.
=	1.	5.
=	0.2	5.

Division by a constant

The reciprocal of 5 is 0.2.

The calculator with the reciprocal key [1/x] will automatically display the reciprocal of the number *in the display* when this key is pressed.

Example: Find the reciprocal of 5.

ENTRY	DISPLAY	STORAGE
5	5.	0.
[1/x]	0.2	0.

Notice in the above simulation table that the storage register was not affected by pressing the reciprocal key. On some calculators the storage will read 5.

Division is an operation which is not commutative.

$$10 \div 5 \neq 5 \div 10$$

However, the answers are *reciprocals of each other.*

$10 \div 5 = 2$; $5 \div 10 = \frac{1}{2}$; $\frac{1}{2}$ is the reciprocal of 2.

Therefore, if we already have a divisor in the display of our calculator, we can divide "upside down" and not have to copy the number in memory or

on a piece of paper, only to reenter it a few seconds later. Dividing "upside down" will give the reciprocal of the proper answer. By immediately pressing the [1/x] key, the proper answer would be displayed.

Example: $\frac{15}{2+3}$ = (15 is dividend) (2 + 3 is divisor)

ENTRY	DISPLAY	STORAGE
2	2.	0.
[+]	2.	2.
3	3.	2.
[÷]	5.	5.
15	15.	5.
[=]	0.3333333	0.3333333 (or 15)
[1/x]	3.0000003	3.0000003 (or 0.3333333)

divisor

dividend

reciprocal of proper answer

Therefore, $\frac{15}{2+3}$ = 3.0000003 (?) We know that 15 ÷ 5 = 3, not 3.0000003. This slight inaccuracy was caused by the fact that 3 ÷ 15 is not exactly 0.3333333. An infinite number of 3's were dropped beyond the last 3 indicated in the display.

There is another approach to the problem above. Instead of dividing the divisor by the dividend and then finding the reciprocal of this result, we could view division as multiplication by the reciprocal of the divisor.

Therefore, $\frac{15}{5} = 15 \cdot \frac{1}{5} = 3.$

Example: $\frac{15}{2+3}$ = ?

ENTRY	DISPLAY	STORAGE
2	2.	0.
[+]	2.	2.
3	3.	2.

(Continued next page)

	ENTRY	DISPLAY	STORAGE
	[=]	5.	5.
Take reciprocal of divisor and multiply.	[1/x]	0.2	5.
	[X]	0.2	0.2
	15	15.	0.2
	[=]	3.	3.

Notice that we obtained the exact answer this time. This is not due to the procedure followed, but to the fact that the reciprocal of 5 was an exact number, 0.2, not an infinite one.

This second method is possibly a little more reliable to use than the first, since you are not likely to forget to key the reciprocal. With the first method, after pressing the [=] key, you might forget to press the [1/x].

Example: $\frac{1}{2} \div \frac{1}{3} = \ ?$

Notice that the numerators of these fractions are 1.

	ENTRY	DISPLAY	STORAGE
	2	2.	0.
Reciprocal of 2 = ½.	[1/x]	0.5	0.
Change operation to multiplication.	[X]	0.5	0.5
Reciprocal of divisor.	3	3.	0.5
	[=]	1.5	1.5

Example: $927 \div \frac{5}{7} = ?$

ENTRY	DISPLAY	STORAGE
5	5.	0.
[÷]	5.	5.
7	7.	5.
[=]	0.7142857	0.7142857
[1/x]	1.4	0.7142857
[X]	1.4	1.4
927	927.	1.4
[=]	1297.8	1297.8

The best policy here, however, would be for the user to rewrite the problem as $927 \cdot \frac{7}{5}$ before starting, and then work the problem as $927 \cdot 7 \div 5$.

Problems:

1. If x is a given number, then the reciprocal of x is ____ .
2. The product of a number and its reciprocal is _____ .
3. Write the reciprocal of the following (in fractional form):

   a. 5 _____ _____

   b. 6 _____ _____

   c. -14 _____ _____

   d. $\frac{1}{3}$ _____ _____

   e. $-\frac{1}{7}$ _____ _____

   f. $\frac{5}{8}$ _____ _____

   g. $\frac{9}{17}$ _____ _____

   h. $-\frac{5}{9}$ _____ _____

4. Use your calculator to find the reciprocal in decimal form for each number in problem # 3. (Divide out the fractions before pressing the $\boxed{1/x}$ key.

5. Evaluate each of the following, using the $\boxed{1/x}$ key if your calculator has one.

   a. $\frac{2370}{563 + 687} =$ _____

   b. $\frac{5.7654}{0.2571 + 3.1779} =$ _____

   c. $\frac{56.38}{11.87 + 8.13} =$ _____

   d. $\frac{317.9328}{0.4603 + 8.3177 - 6.9508} =$ _____

   e. $\frac{1}{8} \div \frac{1}{13} =$ _____

   f. $\frac{1}{16} \div \frac{1}{3} =$ _____

   g. $435 \div (9^2 + 6) =$ _____

   h. $18653.6 \div (63^2 - 638) =$ _____

   i. $849.71 \div (639 - 850) =$ _____

   j. $0.3711 \div \frac{79}{485}$ _____

6. In a parallel circuit in basic electricity the total resistance (R_T) is obtained by the following formula:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots$$

   where R_1, R_2, etc. stand for each individual resistance.

   If R_1 = 30 ohms, R_2 = 20 ohms, and R_3 = 60 ohms, then R_T = _____ ohms.

   (Caution: your first answer will not be R_T but $\frac{1}{R_T}$.)

7. The capitance of condensers joined in series can be found using the formula

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots\ldots$$

   If C_1 = 2 microfarads and C_2 = 5 microfarads, then C_T = _____ microfarads.

8. In basic electricity, conductance (G) in siemens equals 1 divided by resistance in ohms.

$$G = \frac{1}{R}$$

   If R = 5,000 ohms, then G = _____ siemens.

9. The opposition to the flow of a current by a capacitor is called capacitive reactance, abbreviated X_c. It is expressed in ohms and is calculated from the formula

$$X_c = \frac{1}{2\pi fC}$$

where f = frequency of applied voltage in hertz
C = capacitance in farads

Find X_c if f = 120 and C = 0.0001 farads. (Round off answer to the nearest tenth.)

_____ ohms.

10. Tom Seva took a trip by car and recorded the following purchases of gas.

10.8 gal.	$7.05
13.2 "	9.35
6.4 "	4.50
14.9 "	9.23
11.0 "	7.55
10.3 "	6.40
5.6 "	3.70

He covered 1675 miles. What was his average number of miles per gallon? (Round answer off to the nearest tenth of a gallon.)

_____ mpg.

11. What was Tom's average cost per gallon? (Round answer off to the nearest cent.)

_____

12. Tom's Canadian friend Jerry took a trip also and recorded the following purchases of gas.

52.1 litres
36.7
29.3
44.0
55.7
41.9

He covered 2440 kilometres. What was his average number of kilometres per litre? (Round off answer to the nearest tenth of a kilometre.)

_____ km/L

13. Rock A. Feller had shares in the Touchstone Mutual Fund. With the ups and downs of the stock market, the value of the shares fluctuated also. On the first day of each month, Mr. Feller sent the mutual fund a check for $200, to buy as many shares as the money would buy on that day. At the end of six months Mr. Feller tallied up his purchases, which were:

January	23.22 shares	April	24.16 shares
February	21.75 shares	May	25.21 shares
March	20.14 shares	June	26.97 shares

What was the average cost per share that he had paid during this time (to the nearest whole cent)?

_____

VI – SPECIAL KEYS

Lesson 4 — **Exchange Keys, [X↔Y] [X↔M]**

Objectives:

1. You should be able to explain the function of the exchange keys.
2. You should be able to use the exchange keys to exchange the contents of the display register and the storage register, the display register and the memory register, and vice versa.

Information:

Many calculators have "exchange" keys. These enable the user to exchange or switch the numbers that are located in the display register and the storage register, or the display register and the memory register.

Example

ENTRY	(X REGISTER) DISPLAY	(Y REGISTER) STORAGE
5	5.	29. From prior calculation
[X↔Y]	29.	5.
[X↔Y]	5.	29.

Example

ENTRY	(X REGISTER) DISPLAY	(Y REGISTER) STORAGE	(M REGISTER) MEMORY
17	17.	51.	39. From prior calculation
[X↔M]	39.	51.	17.
[X↔M]	17.	51.	39.

The author used the exchange keys demonstrated in the two previous examples to check every simulation table in this textbook.

In the previous lesson we discussed the fact that division is not commutative. Therefore, when we divided in the "wrong order" (the divisor by the dividend), we obtained a result that was the *reciprocal* of the correct answer. By using the reciprocal key we were able to correct the problem and find the answer.

If we remember that division is a binary operation which takes place between the number in the display register (X) and the number in the storage register (Y), then "wrong order" divisions can be corrected – *just before pressing the* [=] *key* – by using the exchange key, [X↔Y] .

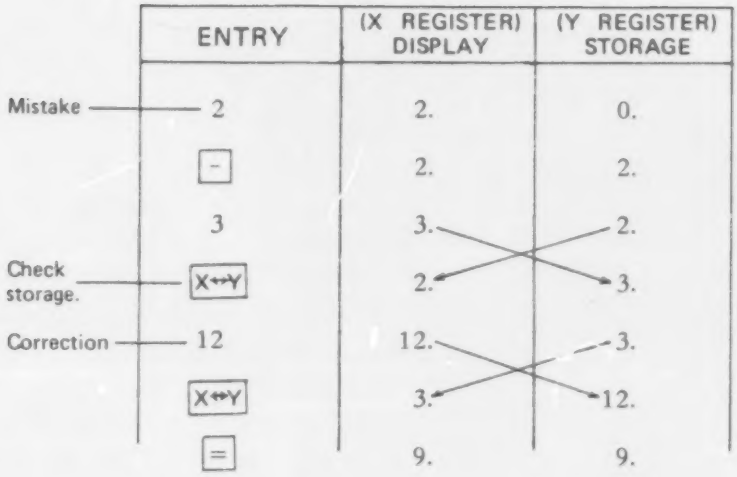
Example: $\frac{15}{2+3} = ?$

ENTRY	(X REGISTER) DISPLAY	(Y REGISTER) STORAGE
2	2.	0.
[+]	2.	2.
3	3.	2.
[÷]	5.	5.
15	15.	5.
[X↔Y] Corrects the order of division.	5.	15.
[=]	3.	3.

Notice that this procedure gives us the exact answer, while the reciprocal-key method sometimes produces slight errors.

The exchange key also allows us to *change or correct numbers in the storage register or the memory register.*

Example: 12 - 3 = ?



It is not uncommon, after working a long problem, to forget exactly what number is in the memory register. The RCL or RM , *recall-memory keys*, will cause the contents in memory to be duplicated in the display, but they will also *cause the number in the display to be erased.* The *exchange key,* X↔M , allows us to exchange the numbers in the display register and the memory register without losing either of these numbers. We would press the key once if we wanted an exchange of numbers. We would press the key twice if we are just checking the number.

Problems:

1. Explain the function of the exchange keys X↔Y and X↔M

2. Complete the following simulation table.

ENTRY	(X REGISTER) DISPLAY	(Y REGISTER) STORAGE	(M REGISTER) MEMORY
12	12.	7.	18.
X↔Y			
X↔M			
X↔Y			
X↔M			
X↔Y			
X↔M			

3. Re-evaluate (a) – (d) and (g) – (j) in problem #5 of the preceding lesson (page 175). Use the exchange key this time. Compare with your answers using the [1/x] key.

4. The relationship between object and image locations for thin lenses can be described by the formula

$$d_i = \frac{d_o \cdot f}{d_o - f}$$

where d_i = distance of image from lens
d_o = distance of object from lens
f = focal length of lens.

If d_o = 12 cm and f = 4.5 cm, then d_i = _____ cm.

5. Re-work problems 10,11,12, and 13 on page 176, using the [X↔Y] key.

6. Carlos kept track of his purchases as he shopped for some items in the Do-It-Yourself-Mart. Find his total bill. Also find the amount *up to that point* at the two times indicated below.

2 @ .69
1 @ 1.49
4 @ .45 [X↔M] = _____
3 @ 1.98
2 @ .39 [X↔M] = _____
10 @ .19
Total = _____

7. Robert had a job making a lawn for a new home. He figured he needed 12 pounds of Scotty's DeLuxe Grass Mixture #1. The small boxes, 1½ pounds each, were on special sale at $2.95 a box. The large boxes, 2½ pounds each, sold at their regular price of $5.25. Using the [X↔M] key if you have one, figure out which size is cheaper. _____

Figure the saving on 12 pounds over the other size. _____

8. Rollo Royce decided which new car he wanted, and looked for a dealer who would give him the best price. Dealer A wanted $4210 for the car, $515 for options, $150 for destination charge, and $75 for dealer prep. Dealer was willing to allow Jay $1350 for his old car.

   Dealer B wanted $3980 for the same car, $490 for options, $150 for destination charge, and $95 for dealer prep. The allowance for Rollo's car would be $1270. Which is the better buy? _____

   How much better? _____

9. Ken Fusion lived near the state line. The moped that he wanted to buy cost $398 plus 5% sales tax from a dealer in his own state. The same machine cost $375 plus 8% sales tax in the other state. Which was the better buy? _____

   By how much? _____

10. Bumburger's Department store was having an appliance sale. They were offering 25% off all major appliances. B. Canny found the model washing machine he wanted; the before-sale price tag read $299.95. Before buying it, he checked with a small dealer who was offering the same model at $275, less 15%. Which was the better buy? _____ By how much? _____

11. A self-employed carpenter paid his estimated income tax in four installments and then "settled up" with the Internal Revenue Service when he filed his return in April. His four payments were

    2 @ $1125.
    2 @ $1215.

    When he filled out his return, he found that his tax came to $4517. Had he overpaid or underpaid? _____

    By how much? _____

VII – PRESENTATION OF NUMBERS

Lesson 1 — **Significant Figures**

Objectives:

1. You should be able to identify the significant figures of a given number.

2. You should be able to round off a number to a specified number of significant figures.

3. You should be able to round off the answers to addition, subtraction, multiplication, and division problems to the proper number of significant figures.

Information:

In the study of pure mathematics we usually assume that each of the digits in every given number is completely accurate.

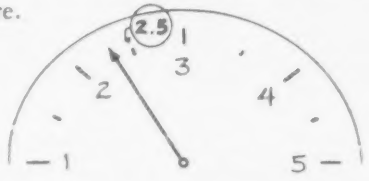
Example: $30 \cdot 42 = 1260$

We would consider 1260 as the correct and accurate answer. We probably would not consider rounding off and stating the answer to this problem as 1000.

The technician and the scientist approach the situation from a slightly different point of view, however. They are generally working with numbers that stand for the result of a *measurement* of some kind. Therefore, the figures can only be as accurate as the limitations of the measuring device allow. For the technician or scientist, accuracy of measurements is expressed by using *significant figures*. The significant figures in a number are those which are *considered to be reliable*.

When reading a measuring instrument, the significant figures are those which are obtained from the printed graduations on the instrument plus one estimated figure.

Example:



Read as "2.3"

When looking at a number to decide how many significant figures it has, we always work from the left (that is, the highest place value) and count, beginning with the first digit that is not zero.

The following rules can serve as guidelines in identifying the significant figures in a given number.

I. **Non-zero digits**

Always significant.

Example: 253.4

All digits significant.

Counting from the left – 4 significant figures.

II. **Zeros**

a. Zeros located *between significant digits* are significant.

Examples: (1) 203 (2) 50.07

Significant

(1) 3 significant figures
(2) 4 significant figures

b. Zeros which are used only as *place holders* are *not* significant. (Either side of the decimal point.)

Examples: (1) 500 (2) 0.005

Not significant *Not significant*

(1) and (2) – 1 significant figure in each number.

c. Zeros which are located on the *right side* of the decimal point *to the right of significant figures* are also significant.

Example: 523.40

Significant

The above number has 5 significant figures.

d. Zeros which are located *to the right of the decimal point*, but not as place holders, are significant. (This makes all zeros to the *left* of this zero also significant, by rule II-a.)

Example: 5600.0 (Zero not needed for decimal-point location)

Significant

The above number has 5 significant figures.

e. Zeros which appear to be placeholders to the left of the decimal point *but are appropriately marked* are considered significant. (All zeros to the *left* of the marked zero are therefore also significant.)

Example: $\underset{\sim}{5}000$ (mark)

Significant *Not significant*

The above number has 3 significant figures.

Examples: Study the following table.

Number	Significant Figures	Rule
9865.1	9, 8, 6, 5, 1	I
4007	4, 0, 0, 7	I, II(a)
6700	6, 7	I, II(b)
0.00516	5, 1, 6	I, II(b)
$18{,}\underset{\sim}{0}00$	1, 8, 0, 0, 0	I, II(e)
$18{,}\underset{\sim}{0}00$	1, 8, 0, 0	I, II(e), II(b)
$18{,}\underset{\sim}{0}00$	1, 8, 0	I, II(e), II(b)
95.0	9, 5. 0	I, II(d)
25.0070	2, 5, 0, 0, 7, 0	I, II(a) (c)
7000	7	I, II(b)
0.0090	9, 0	I, II(c)
13.00	1, 3, 0, 0	I, II(d)
340.0	3, 4, 0, 0	I, II(d)

In Chapter III, Lesson 1 (page 75) we discussed rounding off numbers. (You might wish to review it at this time.) In this lesson we will round off given numbers to an indicated number of significant figures.

Examples: Study the following table.

Number	Round off to: a) Three Significant Figures*	b) Two Significant Figures **
0.05170	0.0517	0.052
67,185.0	67,200	67,000
2.0706	2.07	2.1
50.01	50.0	$\widetilde{50}$
751,000	751,000	750,000
3249.75	3250	3200
5984.1	5980	$\widetilde{6000}$
503.5	504	$\widetilde{500}$

In the operations of addition, subtraction, multiplication, and division, the technician or scientist uses significant figures to identify the reliability of his/her answer. The key thought is that "a chain is as strong as its weakest link."

In *addition and subtraction* a result (answer) depends upon the precision (exactness) of the original numbers. Precision is based upon the *position of the last significant figure to the right.* The answer to an addition or subtraction problem should normally be rounded off so that the precision of the answer is equal to the precision of the *least precise number in the given problem.*

Example: Add the following.

917.308
2.61
5876.871
30.1 ← *Number with least precision – in tenths place.*
2.8994
6829.7884

Therefore, the answer is rounded off to 6829.8, the tenths place.

* Note that the numbers in this column give a rather close approximation of the original numbers.

**Note that the numbers in this column give a very rough approximation of the original numbers.

Example: Subtraction

$$\begin{array}{r} 5200 \\ -\ 1341 \\ \hline 3859 \end{array}$$

Least precise. (Last significant figure in hundreds place.)

Therefore, the answer is rounded off to 3900 – with the last significant figure in the hundreds place.

If you think this last example is unreasonable, follow this argument.

The 5200 (a measurement) could actually represent any number between 5150 and 5249. (Both of these numbers, and all numbers in between, would round off to 5200.) Let us subtract 1341 from both of these numbers.

$$\begin{array}{r} 5150 \\ -\ 1341 \\ \hline 3809 \end{array} \qquad \begin{array}{r} 5249 \\ -\ 1341 \\ \hline 3908 \end{array}$$

The "true" answer could be anywhere between these two numbers. So you see that the answer 3859 "pretends" to an accuracy it does not have. It is better therefore to express the result as 3900, which shows that the answer is an approximation.

The answers in *multiplication and division* are also determined by the number that is least accurate – in this case, the number with the *smallest number of significant figures.* Unless there are special reasons not to, the answer will be rounded off so that it contains the same number of significant figures as *the least accurate number given.*

Example: $2.71828 \cdot 0.031 = ?$

$$2.71828 \cdot \underbrace{0.031}_{\text{Least accurate number – 2 significant figures}} = 0.0842666$$

Therefore, the answer will be rounded to 0.084 (2 significant figures).

Example: $877.625 \div 1.003 = ?$

$$877.625 \div \underbrace{1.003}_{\text{Least accurate number – 4 significant figures}} = 875.$$

Therefore, the answer will be recorded as 875.0 (4 significant figures).

When working out problems on the calculator, always use the entire capacity of the display. Then round off the *final answer* of the problem.

Problems:

1. a. Explain a basic difference in the nature of numbers in pure mathematics and in technical mathematics.

   b. How does this difference affect our treatment of the results (answers) we get when solving problems?

   c. When deciding on the proper number of digits to include in an answer, how do addition and subtraction problems differ from multiplication and division problems?

2. Complete the following table (see example problems, page 184).

Number	Significant Figures	Rule(s)
a. 567.21		
b. 7,500		
c. 7,500.0		
d. 407.03		
e. 352.10		
f. 31,000		
g. 50.001		
h. 30,526.0		
i. 109,000		
j. 0.00370		
k. 0.05		
l. 550		
m. 7,051		
n. 400.0		
0. 30		

3. Complete the following table. (Caution: Be sure that each rounded-off number has the decimal point in the same place as in the original number.)

Number	Round off to: a. Three Significant Figures	b. Two Significant Figures
0.07352		
48,526		
50.835		
413.57		
27.03		
4.009		
3.14159		
700.6		
432,000		
2.007		

4. The following numbers all represent measurements of some sort. Add or subtract each as indicated. Round off the answers to the proper precision.

a.
43.098
7.314
25.02
4.138
132.2

b.
984.2
510.
28.351
4.2
89.89

c.
40,000.0
52.89
407.1
200.
78.92

d.
899.37
124.22
45.765
6.20

e.
905.02
76.5
501.
34.77
4328.22
63.084
766.8

f.
8902.73
- 204.60

g. $\begin{array}{r} 876.2 \\ -\ 638.99 \\ \hline \end{array}$

h. $\begin{array}{r} 0.87021 \\ -\ 0.0030 \\ \hline \end{array}$

i. $\begin{array}{r} 5000 \\ -\ 2388 \\ \hline \end{array}$

j. $\begin{array}{r} 419.703 \\ -\ 386.009 \\ \hline \end{array}$

5. The following numbers all represent measurements of some sort. Multiply or divide as indicated. Round off the answer to the proper accuracy.

a. $56 \cdot 20 =$ _____

b. $2{,}706 \cdot 430 =$ _____

c. $800 \cdot 6.91 =$ _____

d. $0.557 \cdot 7.006 =$ _____

e. $68{,}000 \cdot 16.9 =$ _____

f. $85.0 \cdot 1321.23 =$ _____

g. $1 \div 3 =$ _____

h. $7 \div 9 =$ _____

i. $831.28 \div 51 =$ _____

j. $3.14159 \div 0.006 =$ _____

6. When one or more of the numbers used in computation are *absolute* numbers (not measurements) then the accuracy of the answer should correspond to the accuracy of the measurement number.

a. Divide a 48-minute period into 3 equal parts. _____ min.

b. $1000.00 is to be divided into 6 equal shares. Each share is . . . $ _____

c. Two 12′ boards laid end to end make . . . _____ ft.

d. A 1000-foot racetrack is divided into 3 equal sections. Each section is . . . _____ ft.

e. A 12.50″ metal bar is to have 5 holes drilled at equal distances between them and at each end. Each distance is . . . _____ in.

f. An angle of 310° to be divided into 7 equal parts. Each part is _____ degrees.

Lesson 2 — **Scientific Notation, EE Key**

Objectives:

1. You should be able to express a given number in scientific notation.
2. You should be able to express a number given in scientific notation in ordinary notation.
3. You should be able to use the Enter Exponent key, EE or EEX, to enter exponents.
4. You should be able to change any number in the display to scientific notation.
5. You should be able to add, subtract, multiply, divide, raise to a power, and find the root of a given number expressed in scientific notation.

Information:

Scientific notation is used in scientific work to express very large or very small numbers.

In Chapter II, Lesson 3 (page 66) we discussed exponents and bases. Remember that an exponent tells us how many times to use the base as a factor. For example,

$$10^1 = 10$$
$$10^2 = 10 \cdot 10 = 100$$
$$10^3 = 10 \cdot 10 \cdot 10 = 1{,}000$$
$$10^4 = 10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$$
$$10^5 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 100{,}000$$
$$10^6 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 1{,}000{,}000$$

In Chapter VI, Lesson 3 (page 170) we discussed the reciprocal of a number. Remember that if x is a given number (not zero), then $\frac{1}{x}$ is the reciprocal of x. In algebra $\frac{1}{x}$ is often expressed as x^{-1}. For example,

$$10^{-1} = \frac{1}{10} = 0.1$$
$$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$$
$$10^{-3} = \frac{1}{10^3} = \frac{1}{1{,}000} = 0.001$$

$$10^{-4} = \frac{1}{10^4} = \frac{1}{10{,}000} = 0.0001$$

$$10^{-5} = \frac{1}{10^5} = \frac{1}{100{,}000} = 0.00001$$

$$10^{-6} = \frac{1}{10^6} = \frac{1}{1{,}000{,}000} = 0.000001$$

When expressing a number in **scientific notation**, we write the number as the product of the significant figures of the number times an appropriate power of ten. The decimal point is always placed *immediately to the right of the first significant digit.*

Examples: Numbers written in scientific notation.

a. 2.583×10^5
b. 7.32×10^3
c. 6.011×10^2
d. 3.07×10
e. 4.1×10^{-3}
f. $8. \times 10^{-2}$
g. 6.03×10^{-1}

One important point to remember when we wish to express a number in scientific notation is that the number and its expression in scientific notation are *equal.* We do not "change" the number, we simply change *the way in which the number is expressed.*

Example: Express 2538.2 in scientific notation

In order to express a given number in scientific notation:

a. Move the decimal point from its given position *to the immediate right of the first significant figure.* Keep all significant figures.

2.538.2

b. Use the result of step (a) as a factor times 10^n, where n is the number of places the decimal point was moved in (a). The exponent n is positive if movement in (a) was to the left, and negative if movement in (a) was to the right.

2.5382×10^3 ← 3 places to the left

Answer: 2.5382×10^3

Check:

$2538.2 \stackrel{?}{=} 2.5382 \times 10^3$

$2538.2 \stackrel{?}{=} 2.5382 \times 1000$

$2538.2 = 2538.2$ ✓

Examples: Express each in scientific notation

a. $27.5 = 2.75 \times 10$ ← *(1 understood)*

Check: $2.75 \times 10 = 27.5$ ✓

b. $0.056 = 5.6 \times 10^{-2}$

Check: $5.6 \times \frac{1}{10^2} = \frac{5.6}{100} = 0.056$ ✓

c. $12{,}000{,}000 = 1.2 \times 10^7$

Check: $1.2 \times 10{,}000{,}000 = 12{,}000{,}000$ ✓

d. $0.000000001 = 1 \times 10^{-9}$

Check: $1 \times \frac{1}{1{,}000{,}000{,}000} = 0.000000001$ ✓

Expressing a number given in scientific notation in ordinary notation has been demonstrated in each of the checks above. Here are some additional examples. The idea is to simply treat this as a multiplication problem.

Examples: a. $2.51 \times 10^5 = 2.51 \times 100{,}000 = 251{,}000$

b. $5.8 \times 10^{-3} = 5.8 \times \frac{1}{1000} = 0.0058$

Note that the positive exponent tells us how many places to move the decimal point to the right, while the negative exponent tells us how many places to move the decimal point to the left.

Many calculators, especially those which are scientifically oriented, are equipped with the Enter Exponent Key, EE . This key enables the user

to enter scientific notation by allowing the entry of exponents. The exponents are usually placed to the extreme right side of the display. The base is always 10.

Example: Enter 2.58×10^{6} *(exponent to be entered)*

ENTRY	DISPLAY	STORAGE
2.58	2.58	0.
EE	2.58 00	0. 00
6	2.58 06	0. 00

Although the calculator doesn't show the base 10, the exponent entered is always in relation to base 10.

Example: Enter 50,000,000 in scientific notation.

$$50{,}000{,}000 = 5 \times 10^{7}$$

ENTRY	DISPLAY	STORAGE
5	5.	0.
EE	5. 00	0. 00
7	5. 07	0. 00

Example: Enter 0.0073 in scientific notation.

$$0.0073 = 7.3 \times 10^{-3}$$

ENTRY	DISPLAY	STORAGE
7.3	7.3	0.
EE	7.3 00	0. 00
3	7.3 03	0. 00
+/-	7.3 - 03	0. 00

Notice that the space between the last digit of the number and the first digit of the exponent is used for the sign of the exponent.

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Many times we may wish to express the number in the display (regardless of how it got there) in scientific notation. On many calculators this is accomplished by multiplying the number in the display by 1×10^0.

$$1 \times 10^0 = 1 \times 1 = 1$$

Example:

ENTRY	DISPLAY	STORAGE
	8053.1	8053.1
[X]	8053.1	8053.1
1	1.	8053.1
[EE]	1. 00	8.0531 03
0	1. 00	8.0531 03
[=]	8.0531 03	8.0531 03

Notice that we could have moved the 8.0531 03 to the display register by pressing the exchange key [X↔Y] immediately after the [EE] .

Example: Express 0.0000201 in scientific notation.

ENTRY	DISPLAY	STORAGE
	0.0000201	0.0000201
[X]	0.0000201	0.0000201
1	1.	0.0000201
[EE]	1. 00	2.01-05
0	1. 00	2.01-05
[=]	2.01-05	2.01-05

When we are adding and subtracting numbers in scientific notation we can only add or subtract those terms which have the same exponent.

Example:
$$\begin{array}{r} 7 \times 10^5 \\ +\,8 \times 10^5 \\ \hline 15 \times 10^5 = 1.5 \times 10^6 \end{array}$$

This will be rounded off to 2×10^6.

If we wish to add or subtract terms that are not given with the same exponent, then we will have to express them with the same exponent before proceeding.

Example:

$$\begin{array}{rlll} \text{Add} & 1.63 \times 10^4 & \rightarrow & 1.63 \times 10^4 \\ & +3.8 \times 10^3 & \rightarrow & 0.38 \times 10^4 \\ \hline & & & 2.01 \times 10^4 \end{array}$$

This would be rounded off to 2.0×10^4.

One of the nice features of the hand calculator is that it handles terms with different exponents in addition and subtraction automatically.

Example:
$$\begin{array}{r} 4.57 \times 10^9 \\ -\,3.061 \times 10^8 \end{array}$$

ENTRY	DISPLAY	STORAGE
4.57	4.57	0.
[EE]	4.57 00	0. 00
9	4.57 09	0. 00
[-]	4.57 09	4.57 09
3.061	3.061	4.57 09
[EE]	3.061 00	4.57 09
8	3.061 08	4.57 09
[=]	4.2639 09	4.2639 09

Therefore, the answer is 4.26×10^9 rounded off.

Multiplication and division do not require terms with the same exponents (as addition and subtraction do). Instead, the rules of algebra are followed:

a. Add exponents for multiplication.

b. Subtract exponents for division.

Example:

a. $5 \times 10^3 \cdot 6 \times 10^5 = (5 \cdot 6) \times 10^{3+5}$
$= 30 \times 10^8 = 3 \times 10^9$

b. $2.8 \times 10^7 \div 7 \times 10^3$
$= (2.8 \div 7) \times 10^{7-3}$
$= .4 \times 10^4 = 4 \times 10^3$

As in addition and subtraction, the calculator handles the procedure automatically.

Example: $\underbrace{731 \times 10^8}_{\textit{Not scientific notation}^*} \cdot 4.3 \times 10^{11} = ?$

ENTRY	DISPLAY	STORAGE
731	731.	0.
[EE]	731. 00	0. 00
8	731. 08	0. 00
[X]	7.31 10	7.31 10
4.3	4.3	7.31 10
[EE]	4.3 00	7.31 10
11	4.3 11	7.31 10
[=]	3.1433 22	3.1433 22

Therefore, the answer is 3.1×10^{22} rounded off.

* Notice how 731 08 was expressed in scientific notation when the multiplication key [X] was pressed (731 08 = 7.31 10).

Example: $87{,}052.6 \cdot 6.78 \times 10^{-5} = ?$

ENTRY	DISPLAY	STORAGE
87052.6	87052.6	0.
[X]	87052.6	87052.6
6.78	6.78	87052.6
[EE]	6.78 00	8.70526 04
5	6.78 05	8.70526 04
[+/-]	6.78-05	8.70526 04
[=]	5.9021662 00	5.9021662 00

Therefore, the answer is 5.90×10^{0} rounded off, = 5.90.

Example: $\dfrac{6.7 \times 10^{8} \cdot 8.01 \times 10^{5} \cdot 4.7 \times 10^{15}}{2.73 \times 10^{3} \cdot 2.46 \times 10^{9}}$

ENTRY	DISPLAY	STORAGE	MEMORY
6.7	6.7	0.	0.
[EE]	6.7 00	0. 00	0.
8	6.7 08	0. 00	0.
[X]	6.7 08	6.7 08	0.
8.01	8.01	6.7 08	0.
[EE]	8.01 00	6.7 08	0.
5	8.01 05	6.7 08	0.
[X]	5.3667 14	5.3667 14	0.
4.7	4.7	5.3667 14	0.
[EE]	4.7 00	5.3667 14	0.
15	4.7 15	5.3667 14	0.
[=]	2.522349 30	2.522349 30	0.

(Cont. next page)

$$\frac{6.7 \times 10^{8} \cdot 8.01 \times 10^{5} \cdot 4.7 \times 10^{15}}{2.73 \times 10^{3} \cdot 2.46 \times 10^{9}}$$

ENTRY	DISPLAY	STORAGE	MEMORY
STO	2.522349 30	2.522349 30	2.522349 30
2.73	2.73	2.522349 30	2.522349 30
EE	2.73 00	2.522349 30	2.522349 30
3	2.73 03	2.522349 30	2.522349 30
X	2.73 03	2.73 03	2.522349 30
2.46	2.46	2.73 03	2.522349 30
EE	2.46 00	2.73 03	2.522349 30
9	2.46 09	2.73 03	2.522349 30
÷	6.7158 12	6.7158 12	2.522349 30
RCL	2.522349 30	6.7158 12	2.522349 30
X⇔Y	6.7158 12	2.522349 30	2.522349 30
=	3.7558429 17	3.7558429 17	2.522349 30

Therefore, the answer is 3.8×10^{17} rounded off.

On most scientific calculators, once the EE key is pressed in a continuous calculation, all intermediate and final answers will be given in scientific notation until the Clear key C is pressed.

Powers of ten are very important in the discussion of the metric system. Here is a table of metric prefixes.

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^{18}	exa	E	10^{-1}	deci	d
10^{15}	peta	P	10^{-2}	centi	c
10^{12}	tera	T	10^{-3}	milli	m
10^{9}	giga	G	10^{-6}	micro	μ
10^{6}	mega	M	10^{-9}	nano	n
10^{3}	kilo	k	10^{-12}	pico	p
10^{2}	hecto	h	10^{-15}	femto	f
10	deka	da	10^{-18}	atto	a

Example: megahertz = 1,000,000 hertz

microfarad = $\frac{1}{1000000}$ farad

In algebra we may wish to raise a number that already has an exponent to a power. In such a case we *multiply exponents.*

Example: $(10^2)^5 = 10^{2 \cdot 5} = 10^{10}$

When we raise a number in scientific notation to a given power, the calculator follows the above rule for us automatically.

Example: $(4.3 \times 10^5)^2 = ?$

ENTRY	DISPLAY	STORAGE
4.3	4.3	0.
[EE]	4.3 00	0. 00
5	4.3 05	0. 00
[X^2]	1.849 11	0. 00

An intermediate answer would be 18.49×10^{10}, which the calculator automatically displayed in scientific notation as 1.849 11. Therefore, the answer is 1.8×10^{11} rounded off.

When we are finding the root of an algebraic expression containing an exponent, we divide the exponent by the root index.

Example: $\sqrt[3]{10^{27}} = 10^{27 \div 3} = 10^9$

Example: Find the square root of 4.9×10^9.

ENTRY	DISPLAY	STORAGE
4.9	4.9	0.
[EE]	4.9 00	0. 00
9	4.9 09	0. 00
[$\sqrt{x}$]	7. 04 =	7. 04

The answer is 7×10^4.

If we solved this problem without a calculator, we would express

$\sqrt{4.9 \times 10^9}$ as $\sqrt{49 \times 10^8}$

$\sqrt{49 \times 10^8} = \sqrt{49} \cdot \sqrt{10^8} = 7 \times 10^{8 \div 2}$

$= 7 \times 10^4$

Problems:

1. Express each of the following numbers in scientific notation (without the aid of your calculator).

   a. 5,000 = _____

   b. 0.000007 = _____

   c. 3,070 = _____

   d. 25,080 = _____

   e. 120,000,000 = _____

   f. 257×10^5 = _____

   g. 0.0531 = _____

   h. 0.0000000000000052 = _____

   i. 220.0 = _____

   j. 76,000 = _____

2. Express each number given in scientific notation in normal form.

   a. 2.58×10^4 = _____

   b. 3.01×10^{-3} = _____

   c. 4×10^8 = _____

   d. 1.2708×10^3 = _____

   e. 1.7×10^{12} = _____

   f. 7.0×10^{-8} = _____

   g. 9.053×10^{-2} = _____

   h. 1.500×10^4 = _____

   i. 5.6×10^{-7} = _____

   j. 2.30×10^6 = _____

3. Enter each of the following numbers in scientific notation into your calculator's display. Use [EE] key.

   a. $8.39 \times 10^{5} =$

   b. $4.004 \times 10^{11} =$

   c. $9.8 \times 10^{-7} =$

   d. $4.7 \times 10 =$

   e. $5.3072 \times 10^{18} =$

   f. $7.63 \times 10^{-35} =$

   g. $1.115 \times 10^{-1} =$

   h. $1.0000000 \times 10^{-99} =$

   i. $9.9999999 \times 10^{99} =$

   } range of many 8-digit calculators with scientific notation

   j. $1 \times 10^{0} =$

4. Enter each of the following into the display of your calculator. Have the calculator express each in scientific notation if possible.

   a. 4,329 =

   b. 0.008306 =

   c. 45 =

   d. 12,000.13 =

   e. 358.77 =

   f. 0.0000008 =

   g. 25,000,000 =

   h. 70.060 =

   i. 79,200 =

   j. 0.825 =

5. Evaluate each of the following. Round off according to the rules for significant figures, assuming all figures are measurements.

a. $9 \times 10^{15} + 8.32 \times 10^{14} + 7.5 \times 10^{16} =$ _____

b. $4.05 \times 10^{7} + 8.2 \times 10^{5} - 9.7 \times 10^{6} =$ _____

c. $8.713 \times 10^{22} + 4.3 \times 10^{8} =$ _____

d. $9.57 \times 10^{32} - 6.988 \times 10^{30} =$ _____

e. $7.632 \times 10^{4} - 9.0 \times 10^{3} =$ _____

f. $4.3 \times 10^{-5} - 9.15 \times 10^{-7} =$ _____

g. $3.401 \times 10^{13} \cdot 7.466 \times 10^{20} =$ _____

h. $5.03 \times 10^{-15} \cdot 9.863 \times 10^{20} =$ _____

i. $4.40 \times 10^{3} \cdot 1.785 \times 10^{8} =$ _____

j. $2.3 \times 10^{15} \div 5 \times 10^{-2} =$ _____

k. $8{,}432 \cdot 8.5 \times 10^{6} \div 3.43 \times 10^{-3} =$ _____

l. $\dfrac{5{,}006{,}000 \cdot 6.83 \times 10^{5}}{7 \times 10^{7}}$ _____

m. $\dfrac{2.6 \times 10^{-4} + 8.2 \times 10^{-2}}{6.37 \times 10^{4}}$ _____

n. $\dfrac{5.31 \times 10^{12} \cdot 6.7 \times 10^{22}}{2.66 \times 10^{5} \cdot 3.8 \times 10^{9}}$ _____

6. Evaluate each of the following. Round off to the proper number of significant figures. Express in ordinary notation.

a. $\dfrac{.0123}{7906} =$ _____

b. $95{,}629 \cdot 87{,}954 =$ _____

c. $\dfrac{(22.36 - 21.005) \cdot 0.00089}{58{,}794{,}698} =$ _____

d. $14{,}000 \cdot 8{,}995.4 =$ _____

e. $(698)^2 \div 0.000108 =$ _____

7. The force between two charged plates of a capacitor is given by the equation:

$$f = \frac{1.59\ KAE^2}{\ell^2 \times 10^{11}}$$

where
- f = force between two similar plates, in ounces
- K = relative dielectric constant
- A = area of one side of one plate in square inches
- ℓ = distance between the plates in inches
- E = electromotive force between plates in volts.

If $K = 1$, $A = 16$, $E = 110$, and $\ell = 0.22$, then $f =$ _____ ounce. (Use scientific notation) (Hint: Depending on your calculator, you may need to calculate ℓ^2 before solving the equation.)

8. Capactive reactance can be obtained using the formula

$$X_C = \frac{1}{2\pi f C}$$

where
- X_C = capacitive reactance
- f = frequency
- C = capacitance in farads.

If $f = 60$ and $C = 35 \times 10^{-6}$, then $X_C =$ _____ ohms. (Round off to the tenths place.)

9. The formula for finding the torque exerted on the armature of a motor is

$$T = \frac{\phi \cdot Z \cdot I_a}{4.25 \times 10^8 \cdot m}$$

where
- ϕ = flux passing through the armature
- Z = number of wires in the armature
- I_a = total armature current
- m = number of parallel paths through the armature.

If $\phi = 1{,}200{,}000$, $Z = 400$, $I_a = 10$, and $m = 2$, then $T =$ _____ lb.-ft. (Round off to the hundredths place.)

10. Solve each for P. Use scientific notation. Round off according to significant figures.

a. $P = \dfrac{\sqrt{0.003654}\ (18.37)^3}{(8.724)^4\ \sqrt[4]{743.8}}$ _____

b. $P = \sqrt{\dfrac{(874.3)(0.03816)(28.53)^3}{(1.754)^4(0.007352)}}$ _____

VII – REVIEW PROBLEMS

1. Addition and subtraction

   a. $4{,}528 + 729 + 15{,}351 =$ _____

   b. $0.0038 + 900.671 + 2.579 =$ _____

   c. $1{,}358{,}291 + 31{,}956 + 9{,}978 =$ _____

   d. $15.0 + 20.2 + 39.9 + 45.6 =$ _____

   e. $39.876 - (-5.001) + 45.777 =$ _____

   f. $5{,}321.01 - 2{,}259.88 =$ _____

   g. $1{,}577{,}671 - (-453{,}212) =$ _____

   h. $0.00053 - (-5.05671) - 0.09955 =$ _____

   i. $-84.7 - 4.96 - (-79.85) =$ _____

   j. $-97{,}382.5 + 47{,}576.8 =$ _____

   k. $45.71 + 8.79 - 13.86 - 73.99 =$ _____

   l. $-579 + 604 + 387 - 913 =$ _____

   m. $4.03 \times 10^5 + 8.97 \times 10^5 =$ _____

   n. $8.307 \times 10^{15} + 6.115 \times 10^{15} =$ _____

   o. $4.1 \times 10^{-8} + 7.5 \times 10^{-8} =$ _____

   p. $9.05 \times 10^{-2} + 1.03 \times 10^{-2} =$ _____

2. Multiplication and Division

   a. $57.8 \cdot 3.9 \cdot 0.03 =$ _____

   b. $1.42 \cdot 373 \cdot 18.75 =$ _____

   c. $98 \cdot 13132 \cdot 5.71 =$ _____

   d. $0.01 \cdot 1.302 \cdot 1{,}529 =$ _____

   e. $49{,}481.6 \div 75.2 =$ _____

   f. $6844.4621 \div 76.93 =$ _____

g. $15.04188 \div 0.03 \div 0.08 =$ _____

h. $-0.906 \cdot (-1.301) =$ _____

i. $25{,}734 \cdot (-0.50) =$ _____

j. $618.784 \div (-9.76) \cdot (-15.5) =$ _____

k. $(-0.16048) \div (-0.236) =$ _____

l. $538.356 \div (-986) =$ _____

m. $0.457 \cdot (-35) \cdot 0 =$ _____

n. $95 \div 19 \div 0 =$ _____

o. $9.55 \times 10^5 \cdot 8.71 \times 10^3 =$ _____

p. $1.03 \times 10^7 \cdot 3.2 \times 10^{18} =$ _____

q. $5.601 \times 10^{-15} \cdot 3.001 \times 10^4 =$ _____

r. $5.418 \times 10^2 \div 8.6 \times 10^3 =$ _____

s. $4.365 \times 10^{-5} \div 9.7 \times 10^2 \cdot 1.08 \times 10^{-11} =$ _____

3. Powers and roots

a. $5.32^2 =$ _____

b. $40.1^2 =$ _____

c. $59^2 =$ _____

d. $11^2 =$ _____

e. $12^2 =$ _____

f. $13^2 =$ _____

g. $14^2 =$ _____

h. $15^2 =$ _____

i. $16^2 =$ _____

j. $17^2 =$ _____

k. $18^2 =$ _____

l. $19^2 =$ _____

m. $20^2 =$ _____

n. $0.1^2 =$ _____

o. $0.2^2 =$ _____

p. $9^3 =$ _____

q. $0.05^3 =$ _____

r. $87^3 =$ _____

s. $4.89^3 =$ _____

t. $42.8^4 =$ _____

u. $3^5 =$ _____

v. $115^3 =$ _____

w. $2^7 =$ _____

x. $2{,}529^3 =$ _____

y. $9{,}999{,}999^2 =$ _____

z. $(1.06)^{25} =$ _____

aa. $\sqrt{974169}$ = _____

bb. $\sqrt{0.0004}$ = _____

cc. $\sqrt{3.44569 \times 10^{7}}$ = _____

dd. $\sqrt[3]{21.952}$ = _____

ee. $\sqrt[4]{133.6336}$ = _____

ff. $\sqrt[5]{7.776 \times 10^{43}}$ = _____

4. Fractions *

a. $\frac{3}{8} + \frac{1}{2} - \frac{1}{4}$ = _____

b. $\frac{4}{5} + \frac{2}{3} + \frac{3}{4}$ = _____

c. $5\frac{1}{8} + 13\frac{3}{5}$ = _____

d. $23\frac{7}{8} - 19\frac{2}{5}$ = _____

e. $(\frac{3}{4})^2$ = _____

f. $271\frac{5}{8} - (-36\frac{1}{5}) - 175\frac{1}{2}$ = _____

g. $\frac{2}{5} \cdot \frac{7}{8}$ = _____

h. $\frac{2}{3} \cdot \frac{4}{5} \cdot \frac{1}{5}$ = _____

i. $75\frac{3}{4} \cdot 19\frac{4}{5}$ = _____

j. $6\frac{1}{4} \cdot (-18.31)$ = _____

k. $250\frac{1}{8} \div 5\frac{3}{5}$ = _____

* Operate with 4 decimal places (when needed); round off to 3 places.

5. Percentage

A. Find the indicated percentage of each of the following.

a. 68% of 35 = _____

b. 400% of 21.3 = _____

c. 3% of 82.5 = _____

d. 13.5% of 73.8 = _____

e. 0.2% of 6.99 = _____

B. Markup. Complete the following table.

Cost	Percentage of Markup	Selling Price
a. $ 5.99	75%	
b. $ 10.99	30%	
c. $ 19.95	100%	
d. $146.50	25%	
e. $ 3.00	18%	

C. Discount. Complete the following table.

List Price	Percent of Discount	Selling Price
a. $ 8.99	60%	
b. $14.95	15%	
c. $2,850	20½%	
d. $22.75	4.3%	
e. $216.25	34%	

D. In a recent year, according to form 1040 of the Federal income tax, if a person's taxable income was between $20,000 and $24,000, then the income tax due was $4,380 plus 32% of the amount over $20,000. Find the tax due for each of the following:

a. Taxable income $22,530 _____

b. Taxable income $21,240 _____

c. Taxable income $20,055 _____

d. Taxable income $23,800 _____

e. Taxable income $20,985 _____

E. If the state sales tax is 5%, find the total price due on each of the following.

a. $73 _____

b. $15.40 _____

c. $2,030 _____

d. $8.95 _____

e. $35.45 _____

6. Order of Operations – Evaluate each of the following.

a. $43 \cdot 13.5 + 5865 \div 69 =$ _____

b. $8.7^3 + 40.06 \cdot 0.4 =$ _____

c. $3.75 \cdot 40 \div 25 \cdot 8.3 =$ _____

d. $7 \cdot 156.2 + 14 - 895 =$ _____

e. $18 - (-65) + 1.3^2 \div 26 =$ _____

f. $8.57\,(4.67 - 2.88 + 9.23) =$ _____

g. $16\,(3384 \div 36 + 6) =$ _____

* h. $\frac{5}{9} + \frac{6}{17} + \frac{11}{20} =$ _____

* i. $\frac{15}{32} + \frac{7}{8} - \frac{7}{20} =$ _____

j. $\frac{\sqrt{42^2 + 3652.96}}{50.24 + 23.36} =$ _____

** k. $6.38\,(8.01 + 15.22)^2 =$ _____

l. $[5.9^2 - (18.2 + 97.9)] \div 5 =$ _____

m. $9.7\,[(20.3 + 8.7)^2 - (0.625 + 23.375)^2] =$ _____

n. $\frac{\sqrt{987^2 + 6.354^2 + 2433^2 + 739^2 + 1628}}{63^3 - 249{,}964} =$ _____

* Round off to the thousandths place
** Round off to the hundredths place.

7. Formulas

Substitute the given values into the formulas. Round off answers according to the number of significant figures in the given data.

a. The speed of a car (in miles per hour) can be found by using the formula

$$S = \frac{(rpm) \cdot T}{R \cdot 168}$$

where S = speed in miles per hour
rpm = revolutions per minute of the crankshaft
T = radius of the tires (in inches)
R = rear-axle ratio

If T = 13 in., R = 4.5, and rpm = 3,000, then S = _____ mph.

b. The air resistance (in pounds) of a car can be found using the formula

$$R = 0.0025 \cdot (mph)^2 \cdot A \cdot D$$

where R = air resistance in pounds
mph = speed of the car in miles per hour
A = frontal area in square feet
D = drag coefficient

If mph = 55, A = 30 square feet, and D = 0.4, then R = _____ lbs.

c. The formula $hp = \frac{FWS}{33{,}000}$ can be used to find the horsepower transmitted by a belt. In the formula,

hp = horsepower
F = pull in pounds per inch of width
W = width in inches
S = the speed of the belt in feet per minute

If W = 15 in., F = 90.0 lb., and S = 5,000 ft/min, then hp = _____.

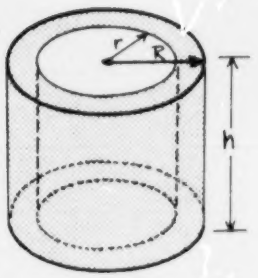
d. The dead distributed safe load on timber of rectangular cross section that is supported at both ends is given by the formula

$$W = \frac{2bd^2K}{L}$$

where W = load in pounds
b = breadth in inches
d = depth in inches
L = length of the span in feet
K = the constant based upon the type of wood

If b = 6 inches, d = 10 inches, L = 16 feet, and K = 1,900, then W = _____ lbs.

e. The amount of material in the wall of a hollow cylinder can be found by using this formula:



$$V = \pi R^2 h - \pi r^2 h$$

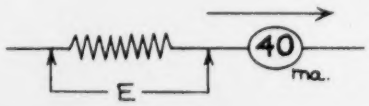
where V = volume between the inside and outside cylinders
R = radius of the outside cylinder
r = radius of the inside cylinder
h = height of the cylinders (Both cylinders have the same height.)

If R = 4.00 inches, r = 3.75 inches, and h = 48 inches, then
V = ____ cubic inches.

8. Scientific Notation

a. Scientific notation is especially useful when working with very large numbers and very small numbers in solving a problem. For example, the voltage drop E across a resistor can be found by multiplying the current flow through the resistor times the resistor value.

$$E = I \cdot R$$



Find E
where R = 5 megohms or 5×10^6 ohms
I = 40 milliamps or 40×10^{-3} amps.

_____

b. The resistance of a wire can be found by the following:

$$R = \rho\left(\frac{l}{A}\right)$$

where R = total resistance in ohms
ρ = "rho" or specific resistance for that material
l = length in feet
A = area in circular mils

Find the total resistance of an aluminum wire whose length is 6500 feet and whose area is 26,250 circular mils. The resistance (ρ) of aluminum wire is 17.

$$R = 17\,\frac{6.5 \times 10^3}{2.625 \times 10^4}$$

R = _____ ohms

ANSWERS TO SELECTED PROBLEMS

I – Introduction – Lesson 1 - Getting Acquainted — Page 12

11.

a.	25	h.	.13	n.	5905	u.	6750
b.	2 5	i.	.013	o.	59.05	v.	675
c.	2.05	j.	.013	p.	19.36	w.	.06
d.	.25	k.	130	q.	19.3	x	38000
e.	250	l.	13000	r.	1900	y.	41.75
f.	2500	m.	13.9	s.	.0025	z.	41.7
g.	13.1			t.	.002		

II – Fundamental Operations – Lesson 1 – Addition and Subtraction — Page 29

5a.

			b.		
Step #1	display	5.	Step #1	display	13.
	storage	0.		storage	0.
Step #2	display	5.	Step #2	display	13.
	storage	5.		storage	13.
Step #3	display	6.	Step #3	display	6.
	storage	5.		storage	13.
Step #4	display	11.	Step #4	display	7.
	storage	11. (or 6.)		storage	7. (or 6.)

6a. 5+8 = ?

ENTRY	DISPLAY	STORAGE
5	5.	0.
[+]	5.	5.
8	8.	5.
[=]	13.	13. (8.) (0.)

b. 25 - 10 = ?

ENTRY	DISPLAY	STORAGE
25	25.	0.
[-]	25.	25.
10	10.	25.
[=]	15.	15. (10. (0.)

c. 12 + 45 - 10 + 3 = ?

ENTRY	DISPLAY	STORAGE
12	12.	0.
[+]	12.	12.
45	45.	12.
[-]	57.	57. (45.)
10	10.	57.
[+]	47.	47. (10.)
3	3.	47.
[=]	50.	50. (3.) (0.)

7. a. 9.
b. 16.
c. 700.
d. 1922.
e. 9297.
f. 680,238.
g. 95.
h. (Clear) 1106.
i. 48.
j. 0
k. (Clear) 462
l. (Clear) 670,526
m. 119.73
n. 44.525
o. 189.87
p. (Clear) 8.83
q. 24,838.75
r. 1.01647
s. (Clear) 100.747
t. 1,634.513
9. 820 ohms
10. 25.1 microfarads
11. $92.877.92
12. $30,924
14. $697
15. T. Lewis – $128.51
L. Garcia – $155.54
16. Balance $160.65
Difference $104.42
17. $197.36
20. (1) 1.715"
(2) 19.24"
(3) 9.71"
(4) 16.275"
(5) 29.6"

II. – Fundamental Operations – Lesson 2 – Multiplication and Division Page 51

3a. Step 1 Display 15.
Storage 0.

Step 2 Display 15.
Storage 15.

Step 3 Display 3.
Storage 15.

Step 4 – Display 45.
Storage 45.

b. Step 1 Display 100.
Storage 0.

Step 2 Display 100.
Storag 100

Step 3 Display 25.
Storage 100.

Step 4 Display 4.
Storage 4.

4 a.

ENTRY	DISPLAY	STORAGE
7	7.	0.
[X]	7.	7.
9	9.	7.
[=]	63	63. (7.) (0.)

b.

ENTRY	DISPLAY	STORAGE
90	90.	0.
[÷]	90.	90.
6.	6.	90.
[=]	15.	15.(6.)(0.)

c.

ENTRY	DISPLAY	STORAGE
5	5.	0.
[X]	5.	5.
3	3.	5.
[X]	15.	15. (3.)
2	2.	15.
[÷]	30.	30. (2.)
15	15.	30.
[÷]	2.	2. (15.)(0.)

5. a. 30
b. 8
c. 11,431
d. 57,969
e. 928
f. 4256
g. 5
h. 35
i. 28
j. 5956
k. 15.5
l. 2054
m. 52.92
n. 1375
o. 40
p. 0.561
q. 500
r. 51.73
s. error indication-division by zero is undefined in mathematics
t. 0

6. 96 watts
7. 1.5 amp
8. a. 200 volts
b. 15 ohms
9. 2.5 ohms
10. a. 68°
b. 98.6°
c. 212°
d. 32°
11. 32 kg
12. 119 rpm
6375 rpm
13. 250 rpm
14. 900 rpm
15. 294 cfm
16. 0.298 in. of water
17. 97.525 cm^2
18. a. 1,327,500 sq. ft.

18. b. 12508.52 m^2
c. 613,850 sq. ft.
d. 483.3593 mm^2
e. 6103.097 cm^2
19. 68201.04 cm^3
20. a. 480,000 cm^3
b. 24.5586 m^3
c. 27.5844 m^3
21. 100,000, 30240
22. 33.6 lb.
23. $31.92
24. $ .37
25. 378 cu. ft. 14 cu. yd.
26. 46656 cu. in.
27. 3648 bricks
28. 350 ft-lb.
29. 83

30.

Item	Total No. of Cans
Niblet Corn 16 oz. Green Giant	192
Cut Wax Beans 16 oz. Green Giant	336
Cream Corn 8 oz. Green Giant	288
Cream Corn 16 oz. Green Giant	768
Green Peas 8 oz. Green Giant	864
Green Peas 16 oz. Green Giant	288
Whole Potatoes 8 oz. Green Giant	480
Whole Potatoes 16 oz. Green Giant	192
Sliced Potatoes 16 oz. Green Giant	264
Tomato Sauce 8 oz. Del Monte	936
Tomato Sauce w/onion 8 oz. Del Monte	1368
Tomato Puree 16 oz. Del Monte	384
Tomato Puree 28 oz. Progresso	288
Total	6648

31.

Lettuce	$ 3.92
Romaine	.78
Chicory	1.40
Cucumbers	2.55
Celery	1.47
Tomatoes	1.95
Total	$ 12.07

32. (1) 23
(2) 20
(3) 29
(4) 12
(5) 28

213

33. 28.7° C

34. 6.7 gal; 7 gal needed

35.

Monday	$ 10.60
Tuesday	9.28
Wednesday	7.29
Friday	13.25
Total	$ 40.42

36. 8 oz. – $.07875 per oz.
12 oz. – .06583 per oz.
18 oz. – .07167 per oz.
Best buy – 12 oz. size

37.

Colson	$ 4.29
Martinez	4.65
O'Neill	7.13
Gomberg	4.43
Korsak	6.84

38. 225 tiles

39. 8.3 lb.; 9 lb. needed

40. 3668.5 bricks; 3700

41. a. 15.2
b. 22.8
c. 13.2
d. 19.8

42. 7956 BTU

43. Brand X – 5.9
Brand Y – 9.3 (would buy)

44. $.157 per mile or 16 cents

45. 43299.7 mm^3

46. 155 rpm

II – Fundamental Operations – Lesson 3 – Powers and Roots **Page 70**

2. a. $5 \cdot 5 \cdot 5 \cdot 5$
b. $17 \cdot 17 \cdot 17 \cdot 17 \cdot 17$
c. $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$
d. $125 \cdot 125 \cdot 125 \cdot 125 \cdot 125$
e. $3 \cdot 3 \cdot 3$

3. a. 529
b. 900
c. 15,625
d. 674,041
e. 169
f. 1.8496
g. 0.0025
h. 2500
i. 811,801
j. 7347.9184

4.

a.	7776	f.	3375
b.	9,663,597	g.	1728
c.	185,193	h.	0.000001
d.	4096	i.	915.0625
e.	3125	j.	256

5.

a.	8	f.	264
b.	15	g.	35.14
c.	17	h.	87.28
d.	24	i.	3698
e.	30	j.	9.257

6.

a.	2	f.	4
b.	3	g.	0.36
c.	2	h.	8
d.	6	i.	16
e.	15	j.	2.54

7.

a.	841	f.	576
b.	3	g.	2
c.	15	h.	.173205
d.	.04	i.	10.4976
e.	10.24695	j.	5

8. 46.208 hp

9. 2.25 watts

10. 976 rpm

11. 0.91 hp

12 (1) 15
(2) 201
(3) 1164
(4) 30.4
(5) 19

13. a. 8.4 cm
b. 4.2 cm
c. 2.8 cm
d. 1.4 cm
e. 11.76 cm^2

14. 58 volts

15. 210.25 sq. ft.

16. 22500 sq. ft.

17. 3.5 ft.

18. 25 in.

19. 208.7 ft.

20. $198.55

3. 15.8911m^2 16m^2

4.

a.	$12.72	f.	$100.59		
b.	92.31	g.	100.60		
c.	6.30	h.	24.01		
d.	10.00	i.	13.15		
e.	15.18	j.	1.00		

5.

a.	$ 60	f.	$421
b.	117	g.	33
c.	2001	h.	14
d.	18	i.	15
i.	152	j.	200

6.

a.	53.25	f.	8,000,000
b.	0.792	g.	76.00
c.	17.2137	h.	100.
d.	259.5	i.	1.
e.	67.80	j.	4532.18

7. 16.524 16.5

3.

a.	0.09375	d.	0.96875
b.	0.21875	e.	0.5
c.	0.59375		

4. 98.80

5.

a.	0.5	i.	0.35
b.	0.333	j.	0.667
c.	0.188	k.	0.5
d.	0.25	l.	0.6
e.	0.625	m.	0.243
f.	0.125	n.	0.79
g.	0.917	o.	0.583
h.	0.6		

6.

a.	1.08	f.	2.55
b.	34.38	g.	171.25
c.	2.4	h.	77.99
d.	0.29	i.	5.2
e.	3.88	j.	9.8

7. $9357.80

8. 124.375 inches; 10.36 feet

9. Size 8 – 316.25
10 – 525.
12 – 781.25
14 – 593.75
16 – 357.5
2573.75

10. 6.4 pieces (or 7)
11. 10.8 (or 11)
12. $21.12

4.

a.	25%	f.	37.5%	k.	5.25%
b.	5%	g.	80%	l.	79%
c.	3.12%	h.	10.4%	m.	0.2%
d.	253%	i.	400%	n.	0.04%
e.	6.5%	j.	615%	o.	13%

5.

a.	.625	f.	.03	k.	.033
b.	.5	g.	.015	l.	.001
c.	.25	h.	.025	m	12
d.	.75	i.	5	n.	.375
e.	.875	j.	1	o.	.1

6. a. 22.5
b. 298
c. 1.2
d. 1.13
e. 130
f. 142
g. 1.50
h. 2.4
i. 80
j. 25.37

7. a. $ 28.62
b. 1992.89
c. 62.30
d. 330.00
e. 13.61
f. .67

8. a. $ 299.99
b. 9.72
c. 5.97
d. 3.56
e. 2050.00
f. 13.76

9. a. $288 $292
b. $14888 $14892
c. $ 2441.25

10. Decrease by 3%

11. $24.30
25.52
3.65

12. $2.84
3.61
6.34
4.11
4.31

13. 28 mpg

14. 80%
72%
84%
52%
100%

15. Get 76% of mpg at 60mph compared with 40mph.

16.

	Toyota Corolla		Ford Pinto		AMC Gremlin		Pontiac Astre	
40 mph	38	–	28	–	27	–	34	–
50 "	35	- 8%	25	- 11%	24	-11%	31	-9%
60 "	29	-24%	23	- 18%	20	-26%	27	-21%

a. Toyoto b. Pinto c. Toyoto
d. Even though the *percent decrease* for the Pinto was smaller, the *base figure* was also smaller; therefore the mileage was not as good as for the Toyoto.

17. Creme de Menage (40%)
No. It started out from a higher *base*.

18. 85.7%

19. 25%

20. 6.67%

21. Colonial Tavern. Although the increase was a smaller percent, the base on which it was figured was larger.

22. Operator C
Operator B

23. 91.3%

24. 17.4%

25. 41%

26. $9420

27. a. 135.7%
b. 59.7%

28. 45%

29. $129.15

30. a. 1.3125%
b. $105.35

31. $4204.20

32. $876.16
57¼%

33. .331
33.1%

34. 24.9%

35. 12.7%

36. $2894.37

3.

a.	b.
100	50.8
28.15	14.3002
3.75	1.905
525	266.7
15.7	7.9756

4.

a.	b.
0.667	0.033
1.852	0.091
5.296	0.26
0.027	0.001
76.741	3.767

7.

a.	$2.39	f.	.42
b.	1.13	g.	2.96
c.	.57	h.	1.09
d.	1.00	i.	3.54
e.	1.52	j.	1.50

8.

a.	31.96	f.	59.92
b.	12.40	g.	26.00
c.	19.80	h.	8.39
d.	14.40	i.	10.36
e.	35.88	j.	16.00

9.

a.	4.8 mm
b.	2.2 mm
c.	3.6 mm
d.	0.3 mm
e.	7.9 mm

10.

Food	75.6°
Housing	90°
Clothing	54°
Transportation	43.2°
Medical	25.2°
Recreation	36°
Savings	21.6°
Misc.	14.4°

11. and 12.

	Current Rate	7.5% Increase	6% Increase	4.5% Increase	Total Increase
a.	3.49	3.75	3.98	4.16	19.2%
b.	3.78	4.06	4.30	4.49	18.8%
c.	4.05	4.35	4.61	4.82	19.0%
d.	5.75	6.18	6.55	6.84	19.0%
e.	4.82	5.18	5.49	5.74	19.1%

13.

Nitrogen	160 kg	Iron	2.6 kg
Phosphorus	44 kg	Manganese	0.4 kg
Potassium	266 kg	Boron	0.4 kg
Calcium	160 kg	Zinc	0.36 kg
Magnesium	40 kg	Copper	0.08 kg
Sulphur	114 kg	Molybdenum	0.08 kg

10.

a.	1500	d.	15,695
b.	350	e.	4559
c.	1485	f.	23,589

11.

a.	17
b.	69.8
c.	8216

12.

	Regular Pay	Overtime Pay	Gross Pay
a.	152.00	–	152.00
b.	210.00	–	210.00
c.	142.45	–	142.45
d.	170.00	95.63	265.63
e.	180.00	135.00	215.00
f.	162.75	–	162.75
g.	240.00	9.00	249.00
h.	338.00	38.03	376.03
i.	280.00	26.25	306.25
j.	138.00	7.76	145.76

13.

	Regular Pay	Overtime Pay	Gross Pay
a.	154.00	–	154.00
b.	128.38	–	128.38
c.	220.00	–	220.00
d.	274.00	–	274.00
e.	140.00	5.25	145.25
f.	162.00	33.41	195.41
g.	250.00	63.28	313.28
h.	146.00	38.33	184.33
i.	146.32	–	146.32
j.	130.00	43.88	173.88

14. $12.05

15. 12 oz. size (.066 vs. .073 per oz.)

16. 19.3 mpg

17. 2.7

18. $1307.00

19. $2563.75

20. $6.39

21.
$ 9.18
9.75
6.15
6.00
5.70
36.78

22. $1530.29

23. $34.75

24. 6648

25. $12.07

26. 16.275"

27. $5625.07

V – Order of Operations – Lesson 1 – Basic Operations

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3.
a. 18
b. 450.88
c. 5184
d. 295
e. 607.725
f. 933
g. 100
h. 2034
i. 18,252
j. 83.5
k. 300.2
l. 89
m. 12.73
n. 4

4.
a. 68°
b. 98.6°
c. 32°
d. 212°

5.
a. 1130 ft/sec
b. 1144 ft/sec
c. 1090 ft/sec

6. Yes
655 lb extra cap.

7. $4300

8. $770.68

3. a.

ENTRY	DISPLAY	STORAGE	MEMORY
16	16.	0.	0.
[×]	16.	16.	0.
[(]	0.	0.	16.
5.	5.	0.	16.
[+]	5.	5.	16.
8	8.	5.	16.
[)]	13.	16.	0.
[=]	208.	13.	0.

b.

ENTRY	DISPLAY	STORAGE	MEMORY
2	2.	0.	0.
[÷]	2.	2.	0.
5	5.	2.	0.
[+]	0.4	5.	0.
[(]	0.	0.	0.4
1	1.	0.	0.4
[÷]	1.	1.	0.4
4	4.	1.	0.4
[)]	0.25	0.4	0.
[+]	0.65	0.25	0.
[(]	0.	0.	0.65
3	3.	0.	0.65
[÷]	3.	3.	0.65
8	8.	3.	0.65
[)]	0.375	0.65	0.
[=]	1.025	0.375	0.

c.

ENTRY	DISPLAY	STORAGE	MEMORY
200	200.	0.	0.
[-]	200.	200.	0.
136	136.	200.	0.
[=]	64.	136.	0.
[√x]	8.	136.	0.
[÷]	8.	8.	0.
[(]	0.	0.	8.
71	71.	0.	8.
[-]	71.	71.	8.
63	63.	71.	8.
[)]	8.	8.	0.
[=]	1.	8.	0.

d.

ENTRY	DISPLAY	STORAGE	MEMORY
13	13.	0.	0.
[+]	13.	13.	0.
29	29.	13.	0.
[X]	42.	29.	0.
[(]	0.	0.	42.
215	215.	0.	42.
[+]	215.	215.	42.
197	197.	215.	42.
[)]	412.	42.	0.
[=]	17304.	412.	0.

4. a. 328,538
b. 53,129.4
c. 54
d. 284
e. 512
f. 1.87
g. 0.49
h. 5
i. 0.2
j. 16,974.593
k. 208163.69 or 208163.70 (depending upon whether your calculator rounds off or not.)
l. 1063.3908
m. 361
n. 54,579
o. 3998
p. 358.5
q. 7.515
r. 4.105

5. a. 44.32
b. 311.031
c. 2493
d. 193.34
e. 139.62
f. 4
g. 59,776,471
h. 0.5
i. 1,000,000

6. a. 0°
b. 20°
c. 37°
d. 100°

7. 1980mm²

8. 0.3 or 30%

9. $1343.92

10. $129.15

11. a. 1.3125%
b. $105.35

12. $231.95

13. $261

14. 84

15. .000019 16. 5.2 ohms 17. 1.25 ohms 18. 4.9 ohms

19. 13.85 or 14 20. 3.91 or 4 gals. 21. 0.0275" per inch

VI – Special Keys – Lesson 1 – [π] Key **Page 152**

2 a. $\frac{355}{113}$

b. (1) $\frac{22}{7}$ - [π] = .00126449

(2) $\frac{220}{70}$ - [π] = .00126449

(3) [π] - $\frac{223}{71}$ = .00074758

(4) $\frac{355}{113}$ - [π] = .00000027

3. a. 1.5707963
b. 3.1415927
c. 4.712389
d. 6.2831854
e. 83.754233
f. 58.259366

4. a. 75.4 mm; 113.1 mm; 12.9 cm; 6.4 cm; 518.4 cm
b. 226.2 mm; 239.4 mm; 125.7 mm; 77.3 cm; 44.3 cm

5. a. 3981.53 sq. in.
b. 0.85 cm^2
c. 1075.21 m^2
d. 193.10 sq. ft.
e. 119,471.31 m^2

6. a. 376.99 mm^2
b. 125.66 mm^2
c. 74.39 cm^2
d. 5,874.78 mm^2
e. 22,455.27 m^2

7. 153,152.6 sq. ft. 8. 1590.3 ft. 9. 3.5 sec.

10. 113.1 ft./sec. 11. 377.0 ampere-turns 12. 1.3 hp

13. 1.63 ft./min. 14. 10.3 hp 15. 60,318.6 in./min.

16. 71,103.0 cm^3 17. 275.23 cm^2 18. 15,793.25 m^3

19. 263.5 cm^2 20. The square, with an area of 256 sq. in; area of circle is 254.5 sq. in.

VI – Special Keys – Lesson 2 – Negative Numbers and Sign-Change Key **Page 166**

6.

ENTRY	DISPLAY	STORAGE
5	5.	0.
[+]	5.	5.
3	3.	5.
[+/-]	-3.	5.
[=]	2.	2.

7.

ENTRY	DISPLAY	STORAGE
7.	7.	0.
X^2	49.	0.
+	49.	49.
10	10.	49.
=	59.	59.
+/-	- 59.	59.
+	- 59.	-59.
81	81.	- 59.
=	22.	22.

8. Yes. Minus a negative is the same as plus the positive of the same number.

9. a. 11.14
b. 11.14
c. 4414.776
d. 4414.776
e. 0.006
f. 0.006
g. -234
h. -234
i. -281.912
j. -281.912

12. a. -825
b. 63,627
c. 1324.4293
d. -1157.144
e. -32.24864
f. -72.93
g. -5287
h. -687.56
i. 40,000
j. 45.167
k. 47,000
l. -102.4

13. a. -33.91
b. -47.2
c. 6.965
d. 481
e. 350
f. 3.6
g. -88,504
h. -60
i. 308.32
j. -1796.803
k. 1156.6241

14. 0.2 or -2

15. Yes; $19.22 left in account

16. $236.23

17. 3596 lb.

18. No; $2.15 short

19. 96 votes

VI – Special Keys – Lesson 3 – Reciprocal Key **Page 174**

3. a. $\frac{1}{5}$
b. $\frac{1}{6}$
c. $-\frac{1}{14}$
d. 3
e. -7
f. $\frac{8}{5}$
g. $\frac{17}{9}$
h. $-\frac{2}{5}$

4. a. 0.2
b. 0.1666666
c. -0.0714285
d. 3.0000003
e. -7.0000021
f. 1.6
g. 1.8888891
h. -1.8000001

5. Answers may vary very slightly, depending on method used.

a. 1.896
b. 1.6784278
c. 2.819
d. 173.99997
e. 1.625
f. 0.1875
g. 5
h. 5.5998107
i. -4.027061
j. 2.2782734

6. R_T = 10 ohms

7. C_T = 1.4 μF

8. 0.0002 siemens

9. 13.3 ohms

10. 23.2 mpg 11. $.66 12. 9.4 km/L 13. $8.48

VI – Special Keys – Lesson 4 – Exchange Keys

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2.

ENTRY	X REGISTER DISPLAY	Y REGISTER STORAGE	M REGISTER MEMORY
12	12.	7.	18.
X↔Y	7.	12.	18.
X↔M	18.	12.	7.
X↔Y	12.	18.	7.
X↔M	7.	18.	12.
X↔Y	18.	7.	12.
X↔M	12.	7.	18.

3. a. 1.896 (1.896)
   b. 1.6784279 (1.6784278)
   c. 2.819 (2.819)
   d. 174 (173.99997)
   g. 5 (5)
   h. 5.6 (5.5998107)
   i. -4.027061 (-4.027061)
   j. 2.2782735 (2.2782734)

4. d_i = 7.2 cm

6. $4.67
   11.39
   13.29

7. Small boxes
   $1.60

8. Dealer B
   $155

9. "Other" state
   $12.90

10. Bumburger's
    $8.79

11. Overpaid
    $163.

VII – Presentation of Numbers – Lesson 1 – Significant Figures

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2 a. 5, 6, 7, 2, 1; I
  b. 7, 5; I, II(b)
  c. 7, 5, 0, 0, 0; I, II(d)
  d. 4, 0, 7, 0, 3; I, II(a)
  e. 3, 5, 2, 1, 0; I, II(c)
  f. 3, 1, 0, 0; I, II(e), II(b)
  g. 5, 0, 0, 0, 1; I, II(a)
  h. 3, 0, 5, 2, 6, 0; I, II(a), II(d)
  i. 1, 0, 9, 0, 0, 0; I, II(a), II(e)
  j. 3, 7, 0; I, II(c)
  k. 5, I, II(b)
  l. 5, 5, 0; I, II(e)
  m. 7, 0, 5, 1; I, II(a)
  n. 4, 0, 0, 0; I, II(d)
  o. 3; I, II(b)

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3.

	a.	b.
0.07352	0.0735	0.074
48,526	48,500	49,000
50.835	50.8	51
413.57	414	410
27.03	27.0	27
4.009	4.01	4.0
3.14159	3.14	3.1
700.6	701	700
432,000	432,000	430,000
2.007	2.01	2.0

4. a. 211.8
b. 1,617
c. 40,700
d. 1075.56
e. 6675
f. 8698.13
g. 237.2
h. 0.8672
i. 2610
j. 33.694

5. a. 1,000
b. 1,200,000
c. 6,000
d. 3.90
e. 1,090,000
f. 112,000
g. 0.3
h. 0.8
i. 16
j. 500

6. d. 333.3 feet e. 2.08" f. 44°

VII – Presentation of Numbers – Lesson 2 – Scientific Notation **Page 200**

1. a. 5×10^{3}
b. 7×10^{-6}
c. 3.07×10^{3}
d. 2.508×10^{4}
e. 1.2×10^{8}
f. 2.57×10^{7}
g. 5.31×10^{-2}
h. 5.2×10^{-15}
i. 2.200×10^{2}
j. 7.600×10^{4}

2. a. 25,800
b. 0.00301
c. 400,000,000
d. 1,270.8
e. 1,700,000,000,000
f. 0.00000007
g. 0.09053
h. 15,000
i. 0.00000056
j. 2,300,000

5. a. 8.5×10^{16}
b. 3.16×10^{7}
c. 8.713×10^{22}
d. 9.50×10^{32}
e. 6.73×10^{4}
f. 4.2×10^{-5}
g. 2.539×10^{34}
h. 4.96×10^{6}
i. 7.85×10^{11}
j. 5×10^{16}
k. 2.1×10^{13}
l. 5×10^{4}
m. 1.3×10^{-6}
n. 3.5×10^{20}

6. a. 0.00000156
b. 8,411,000,000
c. 0.000000000021
d. 130,000,000
e. 4,510,000,000

7. 6.36×10^{-5} or 0.0000636 ounce

8. 75.8 ohms

9. 5.65 lb.-ft.

10. a. $1.239 \times 10^{-2} = 0.01239$
b. $3.337 \times 10^{3} = 3337$

VIII – Review Problems **Page 204**

1. a. 20,608
b. 903.288
c. 1,400,225
d. 120.7
e. 90.654
f. 3061.13
g. 2,030,883
h. 4.95769
i. -9.81
j. -49,805.7
k. -33.35
l. -501
m. 1.30×10^{6}
n. 1.442×10^{16}
o. 1.16×10^{-7}
p. 1.008×10^{-1}

2. a. 6.7626
b. 9931.125
c. 7,348,404.56
d. 19.90758
e. 658
f. 88.97
g. 6267.45
h. 1.179 (rounded off to the thousandths place)
i. -12867
j. 982.7
k. 0.68
l. -0.546
m. 0
n. error indication-division by zero is undefined in mathematics.
o. 8.32×10^{9}
p. 3.3×10^{25}
q. 1.681×10^{-10}
r. 6.3×10^{-2}
s. 4.9×10^{-19}

3. a. 28.3024
b. 1608.01
c. 3481
d. 121
e. 144
f. 169
g. 196
h. 225
i. 256
j. 289
k. 324
l. 361
m. 400
n. 0.01
o. 0.04
p. 729
q. 0.000125
r. 658,503
s. 116.930169
t. 3355637.8
u. 243
v. 1,520,875
w. 128
x. 1.618×10^{10}
y. 9.999998×10^{13}
z. 4.29187072
aa. 987
bb. 0.02
cc. 5.87×10^{3}
dd. 2.8
ee. 3.4
ff. 6×10^{8}

4. a. 0.625
b. 2.217
c. 18.725
d. 4.475
e. 0.563
f. 132.325
g. 0.35
h. 0.107
i. 1499.85
j. -114.438
k. 44.665

5.	A.	B.	C.	D.	E.
a.	23.8	$10.48	$3.60	$5189.60	$76.65
b.	85.2	$14.29	$12.71	$4776.80	$16.17
c.	2.475	$39.90	$2265.75	$4397.60	$2131.50
d.	9.963	$183.13	$21.77	$5596.	$9.40
e.	0.01398	$3.54	$142.73	$4695.20	$37.22

6. a. 665.5
b. 674.527
c. 49.8
d. 212.4
e. 83.065
f. 94.4414
g. 1600
h. 1.458
i. 0.994
j. 1
k. 3442.86
l. -16.258
m. 2570.5
n. 759

7. a. 52 mph
b. 90 lb.
c. 200 hp
d. 100,000 lb.
e. 290 cu. in.

8. a. 2×10^{5} volts or 200 kilovolts
b. 4.2 ohms

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